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Surface Tension Effect on Harmonics of Rayleigh-Taylor Instability

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Using the method of the parameter expansion up to the third order, we explicitly investigate surface tension effect on harmonics at weakly nonlinear stage in Rayleigh-Taylor instability (RTI) for arbitrary Atwood numbers and compares the results with those of classical RTI within the framework of the third-order weakly nonlinear theory. It is found that surface tension strongly reduces the linear growth rate of time, resulting in mild growth of the amplitude of the fundamental mode, and changes amplitudes of the second and third harmonics, as is expressed as a tension factor coupling in amplitudes of the harmonics. On the one hand, surface tension can either decrease or increase the space amplitude; on the other hand, surface tension can also change their phases for some conditions which are explicitly determined.

Key words: Rayleigh-Taylor instability, Surface tension, Harmonics, Weakly nonlinearity

I. INTRODUCTION

Rayleigh-Taylor instability (RTI) plays an important role in many fields ranging from astrophysics, such as supernova explosion [1, 2], to engineering applications, such as inertial confinement fusion (ICF) [3–10]. RTI driven by gravity was first considered by Rayleigh [11], and then Taylor [12]. From then on, problems related with RTI received much attention, but many aspects of dynamics of the instabilities are still uncertain.

RTI occurs on an interface separating two different fluids when a light fluid supports a heavy fluid in a gravity field or the light fluid accelerates the heavy one. Assuming that the heavy fluid is over the light one in a gravitational field $-g\mathbf{e}_y$, where g is gravitational acceleration, an initial interface perturbation between the two fluids of densities ρ_h and ρ_l ($\rho_h > \rho_l$) is in the form of $\eta(x, t=0) = \varepsilon \cos(kx)$ with perturbation wavenumber $k = 2\pi/\lambda$ and perturbation amplitude $\varepsilon \ll \lambda$, where λ is perturbation wavelength. This interface with RTI will develop with time.

When the typical perturbation amplitude is close to its wavelength, the second and third harmonics are generated successively, and then the perturbation accesses the nonlinear regime. Within the framework of the third-order weakly nonlinear theory [13–16], the evolution interface can be described as

$$\eta(x, t) = \sum_{j=1}^3 \eta_j \cos(jkx)$$

$$= (\eta L + \eta_{3,1}^3) \cos(kx) + \eta_{2,2} \eta L^2 \cos(2kx) + \eta_{3,3} \eta L^3 \cos(3kx) \quad (1)$$

with the linear amplitude of the fundamental mode being $\eta_L = \varepsilon \exp(\gamma t)$, where linear growth rate γ and space amplitudes of other harmonics are

$$\gamma = \sqrt{Agk} \quad (2a)$$

$$\eta_{2,2} = -\frac{1}{2} Ak \quad (2b)$$

$$\eta_{3,1} = -\frac{1}{16} (3A^2 + 1) k^2 \quad (2c)$$

$$\eta_{3,3} = \frac{1}{2} (A^2 - \frac{1}{4}) k^2 \quad (2d)$$

$$A = \frac{\rho_h - \rho_l}{\rho_h + \rho_l} \quad (2e)$$

where A is Atwood number. As can be seen in Eq.(2c), the linear growth of the fundamental mode is reduced by the nonlinear effect from the third order, and second and third harmonics (Eq.(2b) and Eq.(2d)), not being corrected by higher orders and called as linear harmonics, grow exponentially in time. One finds that the phase of the fundamental mode is the same as the phase of the initial perturbation, while the second harmonic has an opposite phase (anti-phase) to the initial one. The third harmonic will vanish when Atwood number $A=1/2$. The phase of the third harmonic depends on Atwood number: when $A < 1/2$ ($A > 1/2$), it has an anti-phase (the same phase).

For linear and early nonlinear stages, surface tension effect on RTI is well known [17, 18]: surface tension can produce a cut-off wave number. Based on the potential flow model proposed by Layzer [19], surface tension effect on nonlinear asymptotic solutions of the bubble

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and spike in RTI was investigated in Refs.[20, 21]. The bubble was formed by the lighter fluid rising through the heavy fluid, and the spike was formed by the heavier fluid penetrating down to the lighter fluid. However, little is known for the weakly nonlinear dynamics of RTI with surface tension. In this work, the weakly nonlinear behaviors of the RTI with surface tension are studied.

II. THEORETICAL FRAMEWORK AND ANALYTIC RESULTS

A Cartesian coordinate system in which x and y are, respectively, along and normal to the undisturbed interface between two fluids is built. Assuming the two layer fluids in a gravitational field to be incompressible and irrotational, we write the governing equation for this system as

$$\Delta\phi_i = \frac{\partial^2\phi_i}{\partial x^2} + \frac{\partial^2\phi_i}{\partial y^2} = 0 \quad (3)$$

where ϕ_i ($i=l$ or h) is the potential function of the fluid. This is known as Laplace equation.

We consider an interface between two incompressible fluids in two dimensions. The upper fluid is heavier than the lower fluid. In this work, only an initial single-mode interface is taken into account. This interface, because of effect of the mode-coupling in RTI, will evolve in multimode perturbation. On the evolution interface $y=\eta(x,t)$, the kinematic condition and the Bernoulli equation are

$$\frac{\partial\eta}{\partial t} + \frac{\partial\eta}{\partial x} \frac{\partial\phi_i}{\partial x} - \frac{\partial\phi_i}{\partial y} = 0 \quad (4)$$

$$\left[\left[\rho \left(\frac{\partial\phi}{\partial t} + \frac{1}{2} |\nabla\phi|^2 + gy \right) + p \right] \right] = 0 \quad (5)$$

where the symbol $[[R]] = R_l - R_h$. The kinematic condition implies the continuity of the normal component of fluid velocity across the interface.

On the evolution interface, the normal stress balance is given by

$$[p] = -\sigma \frac{\frac{\partial^2\eta}{\partial x^2}}{\left[1 + \left(\frac{\partial\eta}{\partial x} \right)^2 \right]^{3/2}} \quad (6)$$

where σ is surface tension of the interface. As a result, the dynamic equation for the evolution interface, given by the Bernoulli Eq.(5) and the normal stress balance

(Eq.(6)), is

$$-\left[\left[\rho \left(\frac{\partial\phi}{\partial t} + \frac{1}{2} |\nabla\phi|^2 + gy \right) + p \right] \right] + \sigma \frac{\frac{\partial^2\eta}{\partial x^2}}{\left[1 + \left(\frac{\partial\eta}{\partial x} \right)^2 \right]^{3/2}} = 0 \quad (7)$$

Hence, the evolution of the interface will be uniquely determined by the kinematic condition Eq.(4) and Eq.(7).

As mentioned above, let the initial perturbation amplitude ε be much smaller than the perturbation wavelength λ (*i.e.*, $\varepsilon \ll \lambda$), then the normalized quantity $\hat{\varepsilon} = \varepsilon/\lambda$ will be a small parameter. Based on the theory of the formal perturbation, the interface displacement $\eta(x,t)$ at the weakly nonlinear stage and the velocity potentials $\phi_i(x,y,t)$ can be expanded into a power series in $\hat{\varepsilon}$ as

$$\eta(x,t) = \sum_{n=1}^N \eta_n(t) \cos(nkx) + O(\hat{\varepsilon}^{N+1}) \quad (8a)$$

$$\phi_i(x,y,t) = \sum_{n=1}^N \phi_{i,n}(y,t) \cos(nkx) + O(\hat{\varepsilon}^{N+1}) \quad (8b)$$

where n and N are integer numbers,

$$\eta_n(t) = \sum_{j=n}^N \hat{\varepsilon} \eta_{\sigma,j,n} \exp(j\gamma_\sigma t) \quad (9)$$

which is the amplitude of the n th-harmonic. For $i=h$,

$$\phi_{i,n}(y,t) = \sum_{j=n}^N \hat{\varepsilon}^j \phi_{h,j,n} \exp(j\gamma_\sigma t) \exp(-nky) \quad (10)$$

for $i=l$,

$$\phi_{i,n}(y,t) = \sum_{j=n}^N \hat{\varepsilon}^j \phi_{l,j,n} \exp(j\gamma_\sigma t) \exp(nky) \quad (11)$$

The perturbation velocity potential, $\phi_i(x,y,t)$, has satisfied Laplace Eq.(3) and boundary condition $\nabla\phi_h|_{y \rightarrow +\infty} = 0$ or $\nabla\phi_l|_{y \rightarrow -\infty} = 0$.

Substituting Eq.(8a) and Eq.(8b) into the Eq.(4) and Eq.(7) and collecting terms of the same power in $\hat{\varepsilon}$ to construct a set of equations, one can solve these equations containing the terms $\hat{\varepsilon}^j$ successively for $j=1,2,\dots$.

The results up to the third-order (*i.e.*, $N=3$ in Eq.(8a) and Eq.(8b)) can be obtained. Here, just the results with regard to the evolution interface are shown as

$$\gamma_\sigma = \sqrt{Agk - \frac{k^3\sigma}{\rho_h + \rho_l}}, \quad (12)$$

$$\eta_{\sigma,2,2} = -\frac{Ak [2Ag\rho_l + (A-1)k^2\sigma]}{4 [Ag\rho_l + (1-A)k^2\sigma]} \quad (13)$$

$$\eta_{\sigma,3,1} = \frac{k^2 [8A^2 (3A^2 + 1) g^2 \rho_l^2 + (A - 1)^2 (6A^2 - 7) k^4 \sigma^2 + A (24A^3 - 24A^2 - A + 1) g k^2 \sigma \rho_l]}{64 [-2A^2 g^2 \rho_l^2 + (A - 1) A g k^2 \sigma \rho_l + (A - 1)^2 k^4 \sigma^2]} \quad (14)$$

$$\eta_{\sigma,3,3} = \frac{k^2 [4A^2 (4A^2 - 1) g^2 \rho_l^2 + (A - 1)^2 (4A^2 - 1) k^4 \sigma^2 + A (16A^3 - 16A^2 + 5A - 5) g k^2 \sigma \rho_l]}{16 [2A^2 g^2 \rho_l^2 - 5(A - 1) A g k^2 \sigma \rho_l + 3(A - 1)^2 k^4 \sigma^2]} \quad (15)$$

To examine whether our results recover those for the case without surface tension, we let $\sigma=0$ in above expressions. It is found that these results will reduce to those (Eqs.(2a)–(2d)). From the linear growth rate (Eq.(12)), one finds that surface tension has the stability effect on the evolution interface. As mentioned in Ref.[20], the interface should be stable if surface tension is larger than a critical threshold,

$$\sigma \geq \sigma_c = \frac{(\rho_h - \rho_l)g}{k^2} \quad (16)$$

Eqs.(13)–(15) show that not only Atwood number A , gravity acceleration g , wavenumber k , but also surface tension σ influences the harmonic amplitudes.

III. SURFACE TENSION EFFECT ON HARMONICS

To conveniently explore surface tension effect on the harmonic evolution, we need to define bond numbers $B_{oh}=g\rho_h/(\sigma k^2)$ and $B_{ol}=g\rho_l/(\sigma k^2)$, respectively, for the heavy fluid and light fluid. It is obvious that surface tension is inversely proportional to bond number. As a result, Eqs.(12)–(15) can be expressed in the form of bond number. At the same time, to compare with the results in classical RTI, we reexpress the interface with the coupling effect of surface tension as

$$\begin{aligned} \tilde{\eta}(x, t) &= \sum_{j=1}^3 \tilde{\eta}_j \cos(jkx) \\ &= (\tilde{\eta}_L + c_{3,1}\eta_{3,1}\tilde{\eta}_L^3) \cos(kx) + c_{2,2}\eta_{2,2}\tilde{\eta}_L^2 \cdot \\ &\quad \cos(2kx) + c_{3,3}\eta_{3,3}\tilde{\eta}_L^3 \cos(3kx) \end{aligned} \quad (17)$$

where $\tilde{\eta}_L = \varepsilon \exp(\gamma_\sigma t)$ is linear amplitude of the fundamental mode and γ_σ is linear growth rate in RTI with surface tension. Here the linear growth rate γ_σ and tension factors are given as below:

$$\gamma_\sigma = \sqrt{Akg - \frac{kg}{B_{oh} + B_{ol}}} \quad (18)$$

$$c_{2,2} = \frac{2AB_{ol} + A - 1}{2AB_{ol} - 2A + 2} \quad (19)$$

$$c_{3,1} = \frac{8A^2 (3A^2 + 1) B_{ol}^2 + A (24A^3 - 24A^2 - A + 1) B_{ol} + (A - 1)^2 (6A^2 - 7)}{8A^2 (3A^2 + 1) B_{ol}^2 + 4(A - 1)A (3A^2 + 1) B_{ol} + 4(A - 1)^2 (3A^2 + 1)} \quad (20)$$

$$c_{3,3} = \frac{4A^2 (4A^2 - 1) B_{ol}^2 + A (16A^3 - 16A^2 + 5A - 5) B_{ol} + (A - 1)^2 (4A^2 - 1)}{4A^2 (4A^2 - 1) B_{ol}^2 - 10A (4A^2 - 1) (A - 1) B_{ol} + 6(4A^2 - 1) (A - 1)^2} \quad (21)$$

It is easily found that when bond number tends to be positive infinity, the linear growth rate $\gamma_\sigma = \gamma$ (see Eqs. (18) and (2a)), and tension factors $c_{2,2} = c_{3,1} = c_{3,3} = 1$. That is to say, when surface tension is not taken into account (*i.e.*, $\sigma=0$), the interface above will reduce to the classical one.

When surface tension is considered, the harmonics will be influenced by linear growth rate and tension factors. It is obvious that surface tension reduces the linear growth rate. For the nonlinear harmonic (*i.e.*, the fundamental mode) and linear harmonics (the second and third harmonics), they are influenced by not only the growth rate γ_σ , but also tension factors $c_{3,1}$, $c_{2,2}$ and $c_{3,3}$.

A. The nonlinear harmonic

From FIG. 1, one finds that the amplitude of the fundamental mode increases to its maximum value and then decreases with time. This is because the linear amplitude is corrected by the third-order (see Eq.(17)). For a given Atwood number, it is found that the larger bond number is, the larger the amplitude of the fundamental mode is. This phenomenon is more obvious for smaller Atwood number (shown in FIG. 1(a)). That is to say, surface tension reduces the amplitude of the fundamental mode: the larger the surface tension is, the slower the amplitude of the fundamental grows. As we know, without surface tension, the feedback from the third-order to the fundamental mode, $\eta_{3,1}\eta_L^3$ with $\eta_L = \varepsilon \exp(t\sqrt{Akg}) > 0$ and $\eta_{3,1} < 0$, is always negative. In other words, the negative feedback from the third-order reduces the fundamental mode, resulting in the nonlinear amplitude being always smaller than its linear one of the fundamental mode. When surface tension is considered, feedback of the third-order to the fundamental mode, $c_{3,1}\eta_{3,1}\tilde{\eta}_L^3$, is negative or positive, depending on tension factor $c_{3,1}$: for $c_{3,1} > 0$, it is negative; for $c_{3,1} < 0$, it is positive.

From the expression of the $c_{3,1}$ (*i.e.*, Eq.(20)), one finds that its numerator and denominator are quadratic functions of bond number. Together with bond number

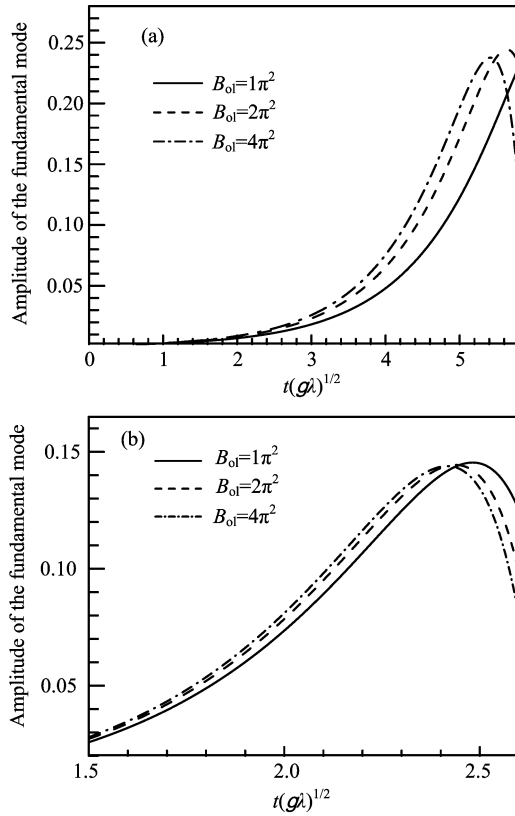


FIG. 1 Amplitudes of the fundamental mode *vs.* time for different bond numbers $B_{ol}=1\pi^2$ (denoted by solid line), $B_{ol}=2\pi^2$ (denoted by dashed line), $B_{ol}=4\pi^2$ (denoted by dot-dashed line) at Atwood numbers (a) $A=0.2$ and (b) $A=0.8$. The amplitude and time are normalized by wavelength λ and gravity acceleration g . The initial amplitude is fixed as $\varepsilon/\lambda=0.001$, and Bond number $B_{oh}=B_{ol}$.

B_{ol} being positive, it is easy to determine conditions of $c_{3,1}>0$ and $c_{3,1}<0$. In FIG. 2, the solid line and dashed line are, respectively, plotted by functions

$$B_{1,1} = \frac{-24A^4 + 24A^3 + A^2}{16(3A^4 + A^2)} + \frac{3\sqrt{48A^6 - 96A^5 + 73A^4 - 50A^3 + 25A^2} - A}{16(3A^4 + A^2)} \quad (22)$$

and

$$B_{1,2} = \frac{1 - A}{2A} \quad (23)$$

When $B_{ol}>B_{1,1}$ (denoted by region I_1) or $B_{ol}<B_{1,2}$ (region III_1), the tension factor $c_{3,1}>0$ makes that the feedback from the third-order to the fundamental mode is negative; when $B_{1,2}<B_{ol}<B_{1,1}$ (region II_1), tension factor $c_{3,1}<0$ makes it positive; when $B_{ol}=B_{1,1}$, the feedback from the third-order to the fundamental mode is zero. Note that tension factor $B_{ol}\neq B_{1,2}$. Accordingly, surface tension can not only reduce the growth of the fundamental mode, but also change the phase of the feedback of the third-order to the fundamental mode.

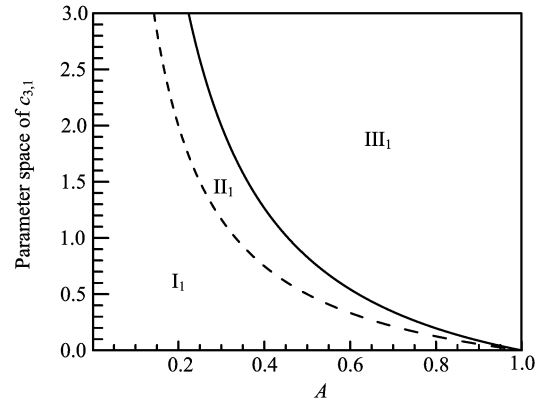


FIG. 2 Parameter space of $c_{3,1}$: region II_1 corresponds to $c_{3,1}<0$, regions I_1 and III_1 correspond to $c_{3,1}>0$.

B. The linear harmonics

Within the framework of third-order weakly nonlinear theory, the second and third harmonics, not corrected by higher-order, will grow in form of $\sim \exp(t\gamma_\sigma)$. Hence, we just need to investigate their tension factors.

C. The second harmonic

For the second harmonic, its amplitude is determined by $c_{2,2}\eta_{2,2}\tilde{\eta}_L^2$ in Eq.(17) where $\eta_{2,2}\tilde{\eta}_L^2<0$ for arbitrary Atwood numbers. As a result, amplitude of the second harmonic is closely related to the tension factor $c_{2,2}$. FIG. 3(a) shows that the value of the $c_{2,2}$ changes with Atwood number. It is found that with the increasing Atwood number, the value changes from -0.5 to 1 . With the increasing bond number, the value is increasing. On the one hand, for the $c_{2,2}$ ranging from 0 to 1 , Bond number strengthens the $c_{2,2}$, denoting that surface tension reduces the $c_{2,2}$; on the other hand, for the $c_{2,2}$ ranging from -1 to 0 , the value is negative. The negative value denotes that the phase of the second harmonic is opposite to that in RTI without surface tension. For this case, surface tension does not reduce but strengthen the $c_{2,2}$. Let $c_{2,2}=0$, one can obtain $B_2=(1-A)/2A$ which is plotted in FIG. 3(b). This figure denotes the parameter space of $c_{2,2}$. When $B_{ol}>B_2$ (denoted by region I_2 in FIG. 3(b)), the $c_{2,2}<0$; when $B_{ol}<B_2$ (denoted by region II_2 in FIG. 3(b)), the $c_{2,2}>0$.

D. The third harmonic

From Eq.(2d), one finds that for $A>0.5$ ($A<0.5$), the third harmonic grows positively (negatively); when $A=0.5$, it vanishes. We show tension factor of the third harmonic, $c_{3,3}$, in FIG. 4(a) for $A<0.5$ and (b) for $A>0.5$, respectively.

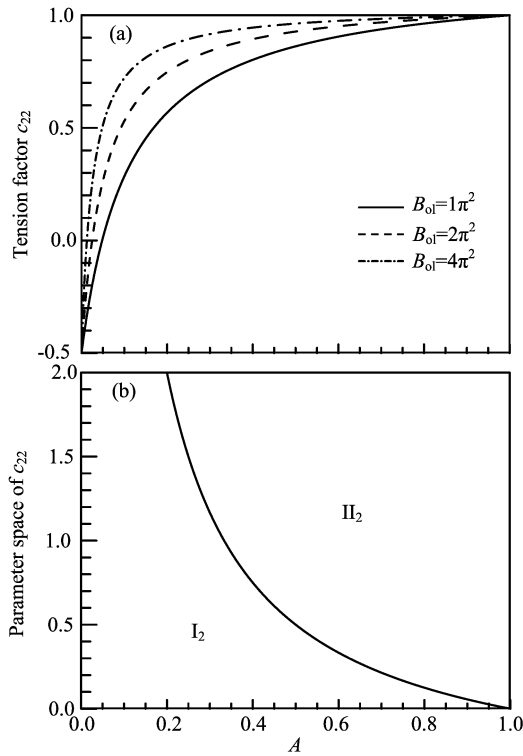


FIG. 3 (a) Tension factor $c_{2,2}$ vs. Atwood number for different Bond number $B_{ol}=1\pi^2$, $B_{ol}=2\pi^2$ and $B_{ol}=4\pi^2$; (b) parameter space of $c_{2,2}$: region I_2 corresponds to $c_{2,2}<0$; region II_2 corresponds to $c_{2,2}>0$.

For $A<0.5$, the value of the tension factor is above zero, showing that the third harmonic has the same phase as the third harmonic in classical RTI. It is found that there is a critical $A_c=0.34$: when $A=A_c$, the $c_{3,3}=1$; when $A<A_c$, the $c_{3,3}<1$; when $A>A_c$, the $c_{3,3}>1$. This denotes that when $A<A_c$, surface tension reduces the amplitude of the third harmonic; while $A>A_c$, surface tension strengthens the amplitude of the third harmonic. The larger surface tension is, the stronger surface tension reduces or strengthens the third harmonic. When $A=A_c$, the effect of surface tension vanishes. For $A>0.5$, the value of the tension factor is less than 1, including negative value. The negative value shows that the third harmonic has an anti-phase comparison to the classical third harmonic.

Parameter space of the $c_{3,3}$ can be shown in FIG. 5. The region I_3 denotes $A<0.5$ for arbitrary Bond numbers, the region II_3 denotes $0.5<A<1$ and $B_{ol}<B_{3,1}$, the region III_3 denotes $0.5<A<1$ and $B_{3,1}<B_{ol}<B_{3,2}$, and the region IV_3 denotes $0.5<A<1$ and $B_{ol}>B_{3,2}$. Here, the $B_{3,1}$ and $B_{3,2}$ are expressed as

$$B_{3,1} = \frac{-16A^4 + 16A^3 - 5A^2 - 3\sqrt{32A^6 - 64A^5 + 33A^4 - 2A^3 + A^2} + 5A}{8(4A^4 - A^2)} \quad (24)$$

$$B_{3,2} = \frac{-16A^4 + 16A^3 - 5A^2 + 3\sqrt{32A^6 - 64A^5 + 33A^4 - 2A^3 + A^2} + 5A}{8(4A^4 - A^2)} \quad (25)$$

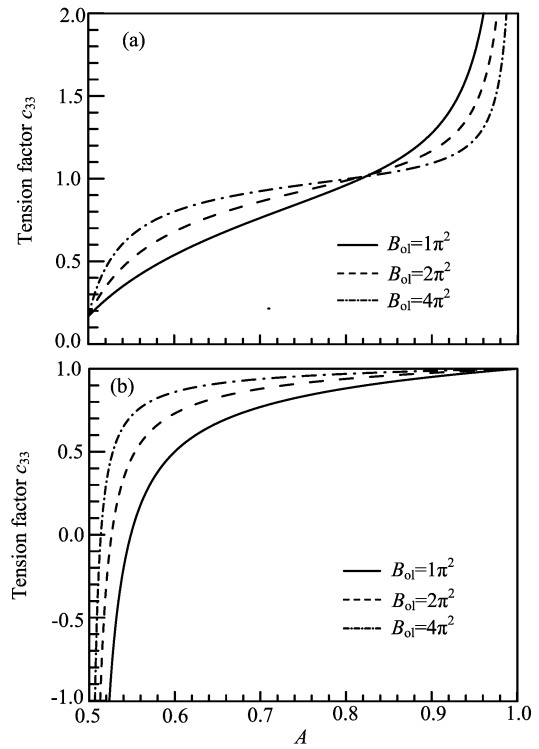


FIG. 4 Tension factor $c_{3,3}$ vs. Atwood number ranging from (a) 0 to 0.5 and (b) 0.5 to 1 for different bond number $B_{ol}=1\pi^2$, $B_{ol}=2\pi^2$ and $B_{ol}=4\pi^2$.

IV. CONCLUSION

The method of the small parameter expansion with nonlinear corrections up to the third order is employed to analytically explore surface tension effect on the harmonics at weakly nonlinear stage in the planar RTI (irrotational, incompressible, and inviscid fluids). When surface tension tends to zero, our results can be reduced to classical ones. Surface tension play an important role in RTI. The influence of surface tension on the harmonics includes two aspects. On one hand, it reduces the linear growth rate (time growth factor). On the other hand, it changes the amplitude and phase of the harmonics (space factor), which is expressed as a single tension factor in this paper.

For the nonlinear harmonic (*i.e.*, the fundamental mode), surface tension reduces its amplitude for arbitrary Atwood numbers: the smaller the Atwood number is, the stronger surface tension reduces the amplitude of the fundamental mode. For the linear harmonics (*i.e.*, the second harmonic and the third harmonic), effect of surface tension appears not only in the linear growth

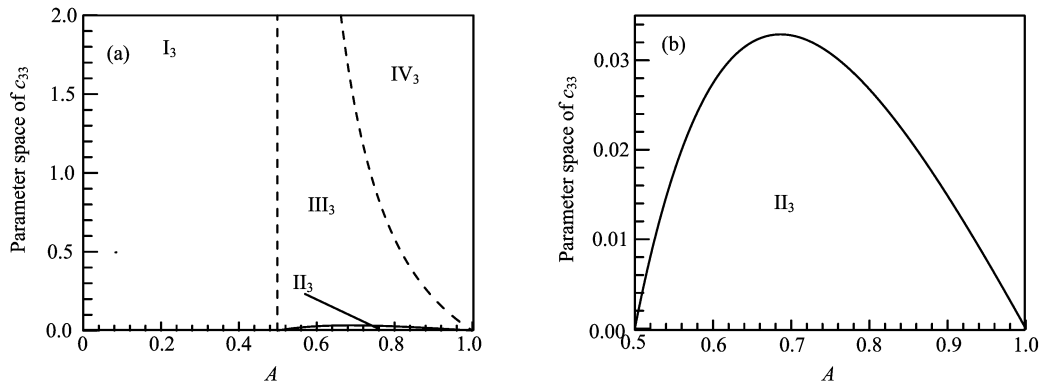


FIG. 5 (a) Parameter space of $c_{3,3}$: regions I_3 , II_3 and IV_3 correspond to $c_{3,3} > 0$ and region III_3 corresponds to $c_{3,3} < 0$. (b) is a local graph for the region II_3 in (a).

rate but also in tension factors. It is found that the tension factor is either positive or negative. The positive tension factor denotes the harmonic has the same phase as that of the corresponding harmonic, and the negative tension factor denotes the harmonic has the opposite phase (anti-phase). For case of either the same phase or the opposite phase, surface tension always reduces the space factor of the second harmonic because the absolute value of the tension factor is less than 1. While for the third harmonic, when Atwood number is less than 0.34, surface tension has a reducing effect; when Atwood number is between 0.34 and 0.5, surface tension has a strengthening effect. For the case $A > 0.5$, surface tension can change the phase of the third harmonic: when the third harmonic has the same phase as that of the corresponding harmonic, surface tension always reduces the space factor, while the third harmonic has the opposite phase, surface tension can either reduce or strengthen the space factor.

V. ACKNOWLEDGEMENTS

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