

ARTICLE

Controlling Coherence Transfer of Dimer Interacting with Independent Environments

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We study the coherence transfer between two pigments (donor and acceptor) of a dimer interacting with two independent environments. By means of a prior weak measurement on the donor and a post measurement, which is either a reversal measurement or a weak measurement, on the acceptor, we present a scheme to optimally control the transfer of the donor's coherence to the acceptor. We construct explicit relationships for the two measurement strengths and the evolution time, by which the coherence degree of the acceptor can approach the maximum value 0.5 at the cost of a decreased probability.

Key words: Dimer, Coherence, Quantum weak measurement, Quantum reversal measurement

I. INTRODUCTION

It has long been believed that the primary processes in photosynthesis took place in a classical way. However, recent experimental results [1–4] indicate that light-harvesting complexes can use quantum-mechanical effects. For example, quantum coherence, a basic concept of quantum mechanics, was shown to play an important role in the excitation energy transfer of light-harvesting complexes [5] and the Fenna-Matthews-Olson complex [6]. It is now recognized that long-lived coherences are responsible for the extraordinarily high efficiency of excitation energy transfer: the absorbed photons are used with efficiencies of $\geq 95\%$ [7] in natural photosynthetic systems. In recent years, the underlying mechanism of quantum coherence facilitating excitation energy transfer has attracted considerable interest in various fields [8–13]. In particular, the efficient photon-to-charge conversion in photosynthetic complexes has inspired considerable efforts to produce artificial systems that take advantage of quantum coherence [14–16]. In practice, the effects of the environment should be considered since coupling of an open system to the external environment is unavoidable and will eventually destroy the coherence possessed by the system. Therefore, designing efficient schemes to realize on-demand control of the system's coherence are important for creating artificial systems. The dimer is a basic component in photosynthetic complexes and consists of two structurally similar units conventionally

called the donor and acceptor. For this simple model in a dissipative environment, transfer of the coherence degree from the donor pigment to the acceptor pigment needs to be improved. Motivated by this demand, in this work, we study the dynamics of the coherence degree of a dimer in a dissipative environment and design an efficient scheme to optimally control it.

In general, the state of a system will collapse by quantum (strong) measurement. However, if the measurement is weak [17–21], *i.e.*, the state is only partially collapsed, it is possible to recover the measured state with a finite probability [22, 23]. The weak measurement is different from the usual projective measurement owing to its unsharpness, and is thus probabilistically reversible. The state of the system after a weak measurement can be recovered via an action called a quantum uncollapsing. Such a reversal measurement has been implemented in experiments for a superconducting phase system [24] and a single-photon system [25]. A prior weak measurement followed by a post reversal measurement has been used to suppress decoherence of a quantum system induced by zero-temperature energy relaxation [26]. This scheme has been experimentally demonstrated in an optical system [27]. A weak measurement without completely collapsing the system moves the system towards its ground state, and the system will therefore be more robust in the later decoherence process. At any desired time, a post reversal measurement can, in principle, recover the system's initial state, but with a finite probability. For the bipartite system, the quantum entanglement contained therein serves as a key resource in realizing quantum information technology. Therefore, several schemes have been proposed to protect entanglement from degradation [28–31] by means of weak and reversal measure-

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ments. In Ref.[29], a scheme using prior weak measurement followed by reversal measurement was proposed and experimentally verified to protect entanglement in amplitude-damping channels. Subsequently, an optimal condition that can maximally improve the system's entanglement has been established for the prior weak measurement strength, the post reversal measurement strength, and the decoherence time [30, 31].

In this work, we use weak and reversal measurements to control the coherence dynamics in a dimer interacting with independent dissipative environments. The dimer is modeled as two two-level atoms A and B coupled together by a dipole-dipole-like interaction: qubit A represents the donor pigment and qubit B being the acceptor pigment. The reservoir a (b) surrounding the pigment A (B) is modeled by a nonperfect cavity. In the following, we present an efficient scheme to maximally transfer the coherence degree from the donor to the acceptor pigment.

II. PHYSICAL MODEL AND THE CONTROLLING SCHEME

The free Hamiltonian of the system and reservoirs is given by

$$\hat{H}_0 = \frac{1}{2} \sum_{i=A,B} \omega_i \hat{\sigma}_+^i \hat{\sigma}_-^i + \sum_{j=a,b} \sum_k \omega_{j,k} \hat{a}_{j,k}^\dagger \hat{a}_{j,k} \quad (1)$$

the interacting Hamiltonian between the pigments with their respective reservoirs can be expressed as

$$\hat{H}_I = \sum_k [(g_{a,k} \hat{\sigma}_+^A \hat{a}_{a,k} + g_{a,k}^* \hat{\sigma}_-^A \hat{a}_{a,k}^\dagger) + (g_{b,k} \hat{\sigma}_+^B \hat{a}_{b,k} + g_{b,k}^* \hat{\sigma}_-^B \hat{a}_{b,k}^\dagger)] \quad (2)$$

and the coupling Hamiltonian of the two pigments reads

$$\hat{H}_C = J(\hat{\sigma}_+^A \hat{\sigma}_-^B + \hat{\sigma}_-^A \hat{\sigma}_+^B) \quad (3)$$

where $\hat{\sigma}_-^i = |0\rangle\langle 1|$ ($\hat{\sigma}_+^i = |1\rangle\langle 0|$) is the lowering (raising) operator of the i th pigment with frequency ω_i , $\hat{a}_{j,k}$ ($\hat{a}_{j,k}^\dagger$) annihilates (creates) a photon in the reservoir j with frequency $\omega_{j,k}$, $g_{j,k}$ denote coupling constant of the pigment and corresponding mode while J that of the two pigments.

Suppose that the initial state of the donor pigment A is $|\psi(0)\rangle_A = \alpha|0\rangle_A + \beta|1\rangle_A$ (with $|\alpha|^2 + |\beta|^2 = 1$), while the acceptor pigment B and both reservoirs are in their ground states, labeled as $|0\rangle_B$ and $|\bar{0}\rangle_j = \prod_{k=1} |0_k\rangle_j$, respectively. In this work, we investigate the coherence transfer from the donor pigment to the acceptor pigment and control of the coherence transfer. The degree of coherence \mathcal{C} can be characterized by the absolute value of the anti-diagonal element of the system's density operator [32]. Generally, the coherence of acceptor

B is less than the initial coherence $|\alpha\beta|$ of the donor A because of dissipation of the reservoirs. To optimally control the coherence transfer, a weak measurement is performed on the pigment A before the interaction, and then quantum measurement reversal is performed on the acceptor B at the desired time $t > 0$. We attempt to determine the optimal strength of the reversal measurement depending on the strength of the prior weak measurement as well as the interaction time.

In practice, a weak measurement is realized by an ideal detector that indirectly detects the system of interest [24–29]. If an excitation is registered in the detector then the system is completely collapsed and the result is discarded. In case the detector is not triggered, which indicates that the system's state is partially collapsed, the system is allowed to continue to evolve. Therefore, the weak measurement with null-outcome is probabilistic, and so is the corresponding control scheme. Mathematically, the weak measurement $\mathcal{W}_X(p)$ with strength p ($0 \leq p < 1$) maps the state of system X as [28, 29]

$$\mathcal{W}_X(p)|n\rangle_X \rightarrow \sqrt{(1-p)^n}|n\rangle_X, \quad n = 0, 1, \quad (4)$$

where p is proportional to the time at which the environment is monitored. After the weak measurement with strength p , the total initial state of the pigments and reservoirs becomes

$$|\Psi(p)\rangle_{AaBb} = \frac{1}{\sqrt{P_1}} (\alpha|0\bar{0}0\bar{0}\rangle_{AaBb} + \beta\sqrt{1-p}|1\bar{0}0\bar{0}\rangle_{AaBb}) \quad (5)$$

where $P_1 = |\alpha|^2 + |\beta|^2(1-p)$ is the null-outcome success probability of this weak measurement. Under the governing Hamiltonian equations (Eqs. (1)–(3)), Eq.(5) will evolve into

$$\begin{aligned} |\Psi(p, t)\rangle_{AaBb} = & c_0(t) |0\bar{0}0\bar{0}\rangle_{AaBb} + \\ & c_1(t) |1\bar{0}0\bar{0}\rangle_{AaBb} + c_2(t) |0\bar{0}1\bar{0}\rangle_{AaBb} + \\ & \sum_k c_{a,k}(t) |01_k0\bar{0}\rangle_{AaBb} + \\ & \sum_k c_{b,k}(t) |0\bar{0}01_k\rangle_{AaBb} \end{aligned} \quad (6)$$

where $|1_k\rangle_j$ denotes one excitation in mode k of reservoir j . Substituting the evolved state Eq.(6) into the Schrodinger equation, give the dynamics equations of evolved coefficients:

$$i\dot{c}_1(t) = \sum_k g_{a,k} \exp[-i(\omega_{a,k} - \omega_0)t] \cdot c_{a,k}(t) + Jc_2(t) \quad (7)$$

$$i\dot{c}_2(t) = \sum_k g_{b,k} \exp[-i(\omega_{b,k} - \omega_0)t] \cdot c_{b,k}(t) + Jc_1(t) \quad (8)$$

$$i\dot{c}_{a,k}(t) = g_{a,k}^* \exp[i(\omega_{a,k} - \omega_0)t] c_1(t) \quad (9)$$

$$i\dot{c}_{b,k}(t) = g_{b,k}^* \exp[i(\omega_{b,k} - \omega_0)t] c_2(t) \quad (10)$$

where we have assumed identical pigments with $\omega_A = \omega_B = \omega_0$. Integrating Eq.(9) and Eq.(10) with the conditions $c_{a,k}(0) = 0$, and inserting their solutions into Eq.(7) and Eq.(8), the equations for $c_1(t)$ and $c_2(t)$ can be obtained:

$$\dot{c}_1(t) = - \int_0^t dt' \sum_k |g_{a,k}|^2 \exp[-i(\omega_{a,k} - \omega_0)(t - t')] \cdot c_1(t') - iJc_2(t) \quad (11)$$

$$\dot{c}_2(t) = - \int_0^t dt' \sum_k |g_{b,k}|^2 \exp[-i(\omega_{b,k} - \omega_0)(t - t')] \cdot c_2(t') - iJc_1(t) \quad (12)$$

where $\sum_k |g_{j,k}|^2 \exp[-i(\omega_{j,k} - \omega_0)(t - t')]$ is the reservoir correlation function $f_j(t-t') = j \langle \bar{0} | A(t) A^\dagger(t') | \bar{0} \rangle_j$, with $A(t) = \sum_k g_{j,k} a_{j,k} \exp[-i(\omega_{j,k} - \omega_0)t]$ and $|\bar{0}\rangle_j$ being the vacuum state of reservoir j . In the limit of a large number of modes, the summation of the reservoir modes is replaced by the integral $\int d\omega S(\omega) \exp[-i(\omega - \omega_0)(t - t')]$, where $S(\omega)$ is called the reservoir effective spectral density. In the continuum limit for the environment, by introducing the correlation function, Eq.(11) and Eq.(12) become

$$\dot{c}_1(t) = - \int_0^t dt' f_a(t-t') c_1(t') - iJc_2(t) \quad (13)$$

$$\dot{c}_2(t) = - \int_0^t dt' f_b(t-t') c_2(t') - iJc_1(t) \quad (14)$$

here we consider a Lorentzian reservoir whose effective spectral density can be modeled as

$$S(\omega) = \frac{W^2 \lambda}{\pi[(\omega - \omega_0)^2 + \lambda^2]} \quad (15)$$

where λ is the half-width at half-height of the cavity field spectrum profile, W is the pigment-environment coupling. Using the Laplace transform and its reverse transform, the solutions of $c_1(t)$ and $c_2(t)$ can be obtained. For convenience, we express $c_1(t)$ and $c_2(t)$ as $c_1(t) = c_1(0)F_1(t)$ and $c_2(t) = c_1(0)F_2(t)$ with $c_1(0) = \beta\sqrt{1-p}/\sqrt{|\alpha|^2 + |\beta|^2(1-p)}$, while $F_1(t)$ and $F_2(t)$ being given in the form of inverse Laplace transformation as

$$F_1(t) = \mathcal{L}^{-1} \frac{s + f(s)}{J^2 + [s + f(s)]^2} \quad (16)$$

$$F_2(t) = \mathcal{L}^{-1} \frac{-iJ}{J^2 + [s + f(s)]^2} \quad (17)$$

where $\mathcal{L}^{-1}\{X(s)\}$ denotes the inverse Laplace transformation of $X(s)$ and $f(s) = W^2/(s + \lambda)$.

As time proceeds, the coherence degree of B becomes nonzero and it can be controlled at any desired

moment through applying another operation on B. It should be noted that the type of the post measurement is situation-dependent: it is a quantum reversal measurement if $|c_2(t)|^2 < 1/2$ and a weak measurement if $|c_2(t)|^2 \geq 1/2$. A quantum reversal measurement $\mathcal{R}_X(p_r)$ with strength p_r ($0 \leq p_r < 1$) on system X corresponds to the map

$$\mathcal{R}_X(p_r)|n\rangle_X \rightarrow \sqrt{(1-p_r)^{n \oplus 1}}|n\rangle_X \quad (18)$$

where \oplus is addition mod 2.

As mentioned above, in the situation when $|c_2(t)|^2 < 1/2$, $\mathcal{R}_B(p_r)$ is performed, by which Eq.(6) is transformed to

$$|\Psi_I(p, t, p_r)\rangle_{AaBb} = \frac{1}{\sqrt{P_I}} \{ c_2(t) |0\bar{0}1\bar{0}\rangle_{AaBb} + \sqrt{1-p_r} [c_0(t) |0\bar{0}0\bar{0}\rangle_{AaBb} + c_1(t) |1\bar{0}0\bar{0}\rangle_{AaBb} + c_3(t) |0\bar{1}0\bar{0}\rangle_{AaBb} + c_4(t) |0\bar{0}0\bar{1}\rangle_{AaBb}] \} \quad (19)$$

$$P_I = |c_2(t)|^2 + (1-p_r)[1 - |c_2(t)|^2] \quad (20)$$

$$|\bar{1}\rangle_a = \frac{1}{c_3(t)} \sum_k c_k(t) |1_k\rangle_a \quad (21)$$

$$|\bar{1}\rangle_b = \frac{1}{c_4(t)} \sum_l c_l(t) |1_l\rangle_b \quad (22)$$

In contrast, in the situation when $|c_2(p, t)|^2 \geq 1/2$, the null-outcome of $\mathcal{W}_B(p_r)$ transforms Eq.(6) to

$$|\Psi_{II}(p, t, p_r)\rangle_{AaBb} = \frac{1}{\sqrt{P_{II}}} \{ \sqrt{1-p_r} c_2(t) |0\bar{0}1\bar{0}\rangle_{AaBb} + c_0(t) |0\bar{0}0\bar{0}\rangle_{AaBb} + c_1(t) |1\bar{0}0\bar{0}\rangle_{AaBb} + c_3(t) |0\bar{1}0\bar{0}\rangle_{AaBb} + c_4(t) |0\bar{0}0\bar{1}\rangle_{AaBb} \} \quad (23)$$

$$P_{II} = (1-p_r)|c_2(t)|^2 + (1 - |c_2(t)|^2) \quad (24)$$

The coherence degree of pigment B for Eq.(19) and Eq.(23) can be obtained by

$$\mathcal{C}_B(p, t, p_r) = \frac{|\alpha\beta|(1-p)^{1/2}(1-p_r)^{1/2}|F_2(t)|}{[|\alpha|^2 + |\beta|^2(1-p)]P_{I,II}} \quad (25)$$

From the control procedure described above, two parameters, p and p_r , can be manipulated at will. The optimal strength $p_r^{(I,II)}$ of the post measurement that maximizes the coherence degree of B at any desired time t can be derived from the conditions

$$\left. \frac{\partial \mathcal{C}_B(p, t, p_r)}{\partial p_r} \right|_{p_r} = 0, \quad \left. \frac{\partial^2 \mathcal{C}_B(p, t, p_r)}{\partial p_r^2} \right|_{p_r} < 0 \quad (26)$$

which gives

$$p_r^{(I)} = \frac{1 - 2|\beta|^2|F_2(t)|^2(1-p) - |\beta|^2 p}{1 - |\beta|^2|F_2(t)|^2(1-p) - |\beta|^2 p} \quad (27)$$

$$p_r^{(II)} = \frac{2(1-p)|\beta|^2|F_2(t)|^2 + p|\beta|^2 - 1}{(1-p)|\beta|^2|F_2(t)|^2} \quad (28)$$

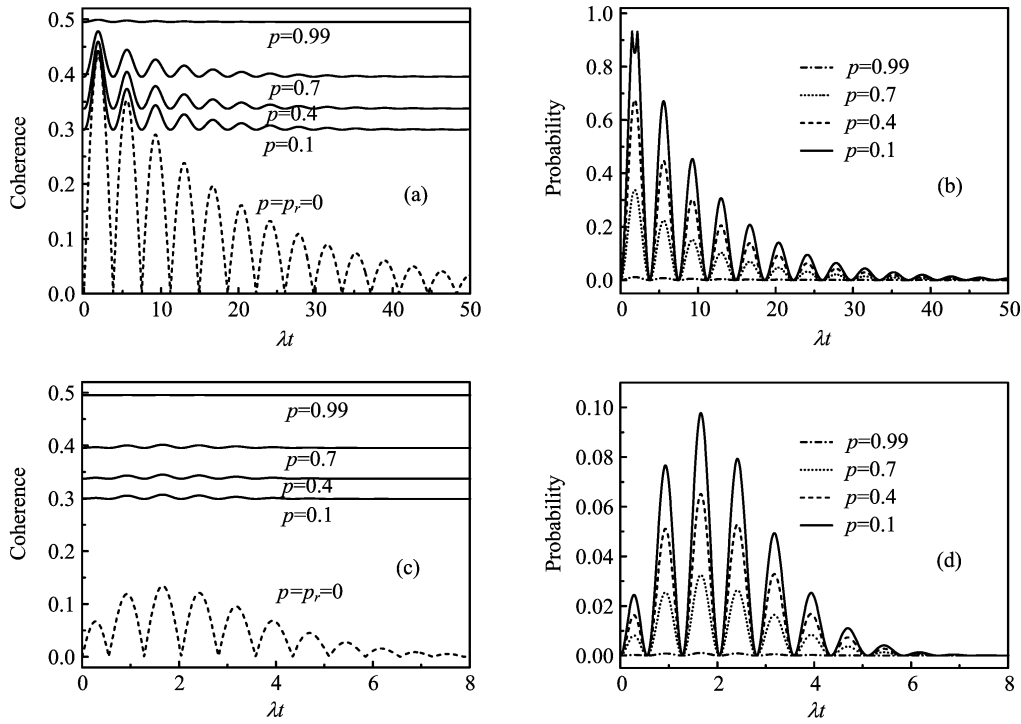


FIG. 1 Optimal coherence $C_B^{\text{opt}}(p, t)$ in Eq.(29) of pigment B versus scaled time λt with respect to different prior weak measurement strength p for (a) $W/\lambda=0.3$ and (b) $W/\lambda=4$. For comparison, we also plot the situation without the controlling scheme (dashed lines in (a) and (c)), *i.e.*, $p=p_r=0$. The corresponding success probabilities are shown in (b) and (d). The coupling strength between the two pigments was chosen as $J/\lambda=0.8$. The initial state of pigment A was prepared as $\alpha=\sqrt{1/3}$ and $\beta=\sqrt{2/3}$.

The corresponding optimal coherence degree $C_B^{\text{opt}}(p, t)$, in accordance with the conditions Eq.(27) and Eq.(28), has the same form

$$C_B^{\text{opt}}(p, t) = \frac{|\alpha|}{2} \sqrt{\frac{1}{1 - |\beta|^2[(1-p)|F_2(t)|^2 + p]}} \quad (29)$$

but the two success probabilities differ:

$$P_I^{\text{opt}} = 1 - p|\beta|^2 - p_r^{(I)}[1 - p|\beta|^2 - |\beta|^2(1-p)|F_2(t)|^2] \quad (30)$$

$$P_{II}^{\text{opt}} = 1 - p|\beta|^2 - p_r^{(II)}|\beta|^2(1-p)|F_2(t)|^2 \quad (31)$$

We observe that for $p \rightarrow 1$ the optimal coherence degree $C_B^{\text{opt}}(p, t) = 1/2$, *i.e.*, the largest coherence of the acceptor is achieved. In this case, however, the success probability tends to zero.

Figure 1 shows the improvement of the coherence degree of the acceptor pigment B under the optimal condition for different strengths (p) of weak measurement and the corresponding success probabilities from our scheme. Figure 1 (a) and (c) show that at any desired time during the evolution, the coherence degree of B, *i.e.*, $C_B^{\text{opt}}(p, t)$ in Eq.(29), is markedly increased by our scheme compared with the situation without control ($p=p_r=0$), for the weak and the strong system-environment couplings. Furthermore, $C_B^{\text{opt}}(p, t)$

increases with increasing p . Because of the presence of the interaction between A and B, the dynamics experience damping oscillations even in the weak system-environment coupling regime (Fig.1(a)). The oscillation amplitude decreases with increasing p because the “stronger weak” measurement makes the coherence remain for a longer time at pigment B and inhibits its transfer between the two pigments. For the strong coupling regime, additional oscillations caused by the environmental memory effect come into play, which interferes with that due to pigment-pigment interactions, resulting in irregular damped oscillations, as seen in Fig.1(c). Interestingly, the oscillations interfering in the regime of strong coupling results in the saturation time being much shorter than that in the regime of weak coupling. Figure 1 (b) and (d) show the corresponding success probabilities. Generally, the probability decreases with increasing p and becomes zero for $p \rightarrow 1$. It is worth noting that the probability exhibits large-amplitude transient oscillations before reaching the steady value. In fact, the coherence is exchanged between the two pigments during the evolution and, therefore, if at the time the post control action is performed the coherence is mostly located at B, then the probability of successfully enhancing the coherence degree of B is large, and vice versa.

III. CONCLUSION

In conclusion, we have studied the coherence dynamics of a dimer interacting with two independent dissipative environments. We designed a control scheme for coherence transfer by means of choosing a suitable combination of quantum weak and quantum reversal measurements. We took into account the situation when only the donor pigment possesses some degree of coherence at the initial time. The scheme aims for optimal transfer of coherence from the donor to the acceptor pigment. For this purpose, a prior weak measurement with strength p is performed on the donor pigment at the time the system starts to evolve ($t=0$). Then, at a desired time during evolution ($t>0$), a post measurement with strength p_r is carried out on the acceptor pigment. However, the post measurement should be chosen as either a quantum reversal or a weak measurement depending on the evolved state of the system at the desired time t . For a given p of the weak measurement strength, we established formula for the post measurement strengths p_r to achieve the optimal on-demand control mentioned above. In theory, when p_r satisfies the established condition, the coherence degree of the acceptor at the desired time t can approach $1/2$ by setting p tend to 1. However, as a rule, for $p \rightarrow 1$ the success probability tends to zero. Finally, we briefly discuss the experimental implementation of the proposed scheme. Indeed, the two main components of the scheme, *i.e.*, the quantum weak and reversal measurements, have been realized in various contexts. A reversal of the weak measurement was demonstrated experimentally in a superconducting phase qubit [24] and in a single-photon qubit [25]. Using all-optical apparatuses, suppression of the decoherence of a single qubit [27] and disentanglement of two entangled qubits [29] via weak measurement and its reversal were also achieved experimentally. As a result, with today's technology, the proposed control scheme could be applied in artificial systems when quantum coherence plays an important role.

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- [1] T. Brixner, J. Stenger, H. M. Vaswani, M. Cho, R. E. Blankenship, and G. R. Fleming, *Nature* **434**, 625 (2005).
- [2] G. S. Engel, T. R. Calhoun, E. L. Read, T. K. Ahn, T. Mancal, Y. C. Cheng, R. E. Blankenship, and G. R. Fleming, *Nature* **446**, 782 (2007).
- [3] H. Lee, Y. C. Cheng, and G. R. Fleming, *Science* **316**, 1462 (2007).
- [4] E. Collini and G. D. Scholes, *Science* **323**, 369 (2009).
- [5] X. Hu and K. Schulten, *Phys. Today* **50**, 28 (1997).
- [6] R. E. Fenna and B. Matthews, *Nature* **258**, 573 (1975).
- [7] Y. C. Cheng and G. R. Fleming, *Annu. Rev. Phys. Chem.* **60**, 241 (2009).
- [8] R. J. Sension, *Nature* **446**, 740 (2007).
- [9] M. Mohseni, P. Rebentrost, S. Lloyd, and A. A. Guzik, *J. Chem. Phys.* **129**, 174106 (2008).
- [10] J. Cao and R. J. Silbey, *J. Phys. Chem. A* **113**, 13825 (2009).
- [11] F. Caruso, A. W. Chin, A. Datta, S. F. Huelga, and M. B. Plenio, *J. Chem. Phys.* **131**, 105106 (2009).
- [12] M. Sarover, A. Ishizaki, G. R. Fleming, and K. B. Whaley, *Nat. Phys.* **6**, 462 (2010).
- [13] S. Tanaka, *Chem. Phys. Lett.* **508**, 139 (2011).
- [14] A. C. Benniston and A. Harriman, *Mater. Today* **11**, 26 (2008).
- [15] M. Escalante, A. Lenferink, Y. P. Zhao, N. Tas, J. Huskens, C. N. Hunter, V. Subramaniam, and C. Otto, *Nano Lett.* **10**, 1450 (2010).
- [16] E. Rozbicki and P. Machnikowski, *Phys. Rev. Lett.* **100**, 027401 (2008).
- [17] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, Cambridge: Cambridge University Press, (2000).
- [18] A. N. Korotkov, *Phys. Rev. B* **60**, 5737 (1999).
- [19] H. S. Goan and G. J. Milburn, *Phys. Rev. B* **64**, 235307 (2001).
- [20] S. Pilgram and M. Buttiker, *Phys. Rev. Lett.* **89**, 200401 (2002).
- [21] A. A. Clerk, S. M. Girvin, and A. D. Stone, *Phys. Rev. B* **67**, 165324 (2003).
- [22] A. N. Korotkov and A. N. Jordan, *Phys. Rev. Lett.* **97**, 166805 (2006).
- [23] Q. Sun, M. Al-Amri, and M. S. Zubairy, *Phys. Rev. A* **80**, 033838 (2009).
- [24] N. Katz, M. Neeley, M. Ansmann, R. C. Bialczak, M. Hofheinz, E. Lucero, A. Connell, H. Wang, A. N. Cleland, J. M. Martinis, and A. N. Korotkov, *Phys. Rev. Lett.* **101**, 200401 (2008).
- [25] Y. S. Kim, Y. W. Cho, Y. S. Ra, and Y. H. Kim, *Opt. Express* **17**, 11978 (2009).
- [26] A. N. Korotkov and K. Keane, *Phys. Rev. A* **81**, 040103(R) (2010).
- [27] J. C. Lee, Y. C. Jeong, Y. S. Kim, and Y. H. Kim, *Opt. Express* **19**, 16309 (2011).
- [28] Q. Sun, M. Al-Amri, L. Davidovich, and M. Suhail Zubairy, *Phys. Rev. A* **82**, 052323 (2010).
- [29] Y. S. Kim, J. C. Lee, O. Kwon, and Y. H. Kim, *Nat. Phys.* **8**, 117 (2012).
- [30] Z. X. Man, Y. J. Xia, and N. B. An, *Phys. Rev. A* **86**, 052322 (2012).
- [31] Z. X. Man, Y. J. Xia, and N. B. An, *Phys. Rev. A* **86**, 012325 (2012).
- [32] T. Baumgratz, M. Cramer, and M. B. Plenio, *Phys. Rev. Lett.* **113**, 140401 (2014).