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Array-enhanced Logical Stochastic Resonance in Coupled Bistable Systems

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We investigate the impact of coupling on the reliability of the logic system as well as the logical stochastic resonance (LSR) phenomenon in the coupled logic gates system. It is found that compared with single logic gate, the coupled system could yield reliable logic outputs in a much wider noise region, which means coupling can obviously improve the reliability of the logic system and thus enhance the LSR effect. Moreover, we find that the enhancement is larger for larger system size, whereas for large enough size the enhancement seems to be saturated. Finally, we also examine the effect of coupling strength, it can be observed that the noise region where reliable logic outputs can be obtained evolves non-monotonically as the coupling strength increases, displaying a resonance-like effect.

Key words: Logic gate, Noise, Coupling, Logical stochastic resonance

I. INTRODUCTION

Over the past few years, it has become increasingly explicit that nonlinearity and noise can cooperate in a dynamical system to produce nontrivial ordered behavior. One of the most famous examples is stochastic resonance (SR) phenomenon, which means that the addition of an appropriate amount of noise sometimes can optimally enhance output response through a delicate interplay between input signal, noise and nonlinearity [1]. SR has received extensive attention over the past two decades, and it has been demonstrated over a variety of physical and neurological systems [2]. On the other hand, as computational devices and platforms tend to shrink in size and increase in speed, we are increasingly encountering fundamental circuit noise characteristics which cannot be suppressed or eliminated [3]. Hence the construction of reliable computational devices under the impact of thermal noise is a subject of great importance.

Inspired by the SR theory, some researchers have realized that the cooperative behavior between a device's noise floor and its nonlinearity can be utilized to design and construct reliable computational devices. Recently, Murali *et al.* employed piecewise linear system with two thresholds to construct a logic gate, and demonstrated that the interplay between its nonlinearity and device noise could yield reliable logical response in the presence of moderate noise [4, 5]. They termed this phenomenon logical stochastic resonance (LSR). The concept of LSR allows one to design a new kind of computational elements, unlike traditional ones, which can work very properly when the noise floor cannot be suppressed. Since this pioneer work, creating reliable logical gates through LSR has become a focal research topic [5–18]. One typical noisy logic gate has been implemented via a cubiclike nonlinear electronic circuit using a linear resistor, a linear capacitor, and four complementary metal-oxide-semiconductor (CMOS) transistors [5]. The circuit is tested through various combination of two logic inputs, and it is found that for very small or large noise the system cannot operate reliably, whereas in a reasonably wide region of moderate noise, the system produces the desired logic outputs consistently. Another example reported by Guerra *et al.* exhibited that the LSR can work very reliably in some nanoscale devices, which had a potential application of designing nontraditional nanomechanical computational devices [8]. The phenomenon of LSR has also been verified in many diverse systems, such as compact three-terminal resonant tunneling diode [9], vertical cavity surface emitting laser (VCSEL) [10], polarization bistable laser [14], synthetic gene networks [15–17], and so on. These studies could be applied to design optical and biological computers.

It is worth noting that in the aforementioned literature the researchers study LSR with single logic gate. Whereas in real computational devices, *e.g.*, integrated circuit (IC), a large number of logic gates are packed with conducting wires into the surface of semiconductor material to carry out complex computation. Coupling between logic gates is a very important factor in constructing computational devices. Moreover, the above logic gate devices are all constructed through noisy non-

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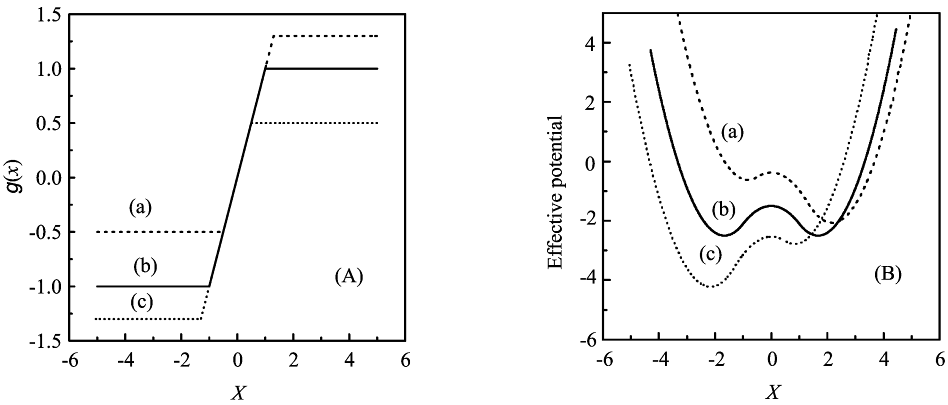

FIG. 1 (A) The piecewise linear function $g(x)$ and (B) the effective potential, it gives rise to, with thresholds (a) $x_l^*=-0.5$, $x_u^*=1.3$, (b) $x_l^*=-1.0$, $x_u^*=1.0$ and (c) $x_l^*=-1.3$, $x_u^*=0.5$. Equal upper and lower thresholds (*i.e.*, $x_l^*=x_u^*$) give rise to symmetric wells. α=1.8, β=3.0, $I_{1,2}$=0

linear systems. It is well known that coupling often plays an important role in nonlinear dynamical systems, and has created a lot of remarkable behaviors, such as inducing dynamical synchronization [19, 20], enhancing SR and coherence resonance (CR) [21–26], inducing system size resonance [27–30], *etc.*

In this work, we focus our attention on the coupled logic gates system, investigate the impact of coupling on the reliability of the logic system as well as the LSR phenomenon. The various cycling combinations of two logic inputs are fed to the system, and the outputs are checked according to the truth tables of fundamental logic operation. The reliability of the logic system is measured by the probability of yielding correct logic outputs. It is shown that for the coupled system, the LSR still exists robustly, *i.e.*, the system can produce desired logic output reliably only in a moderate region of noise intensities (we call this noise region the reliable region). In addition, we find that the reliable region of coupled system is much wider than the single logic gate, which means coupling can significantly improve the reliability of the logic system and thus enhance the LSR effect. This behavior can be termed as array-enhanced logical stochastic resonance. As the size of the coupled system increases, the reliable region initially increases, then reaches a plateau. We have also examined the effect of coupling strength, it can be observed that the size of the reliable region exhibits a resonance-like effect as a function of coupling strength, and when the coupling strength exceeds a certain threshold, the reliable region disappears, which means that the system cannot yield any reliable logical response when the coupling is too strong.

II. THE COUPLED LOGIC GATES

In this work, we investigate the LSR effect in the coupled logic gates system. Each of the N coupled logic gates is described by a general nonlinear dynamical system, which can be implemented via a simple electronic analog circuit [4, 5]:

$$\dot{x} = F(x) + I_1(t) + I_2(t) + D\eta(t) \tag{1}$$

where x denotes the response of the logic gate, $F(x)$ is a generic nonlinear function giving rise to a potential with two distinct energy wells, $I_1(t)$ and $I_2(t)$ are two aperiodic square waves which encode the two logic inputs, $\eta(t)$ is an additive zero-mean Gaussian noise with unit variance, D being the noise intensity.

In this work, we consider the same $F(x)$ as in the Ref.[4], given by

$$F(x) = -\alpha x + \beta g(x) \tag{2}$$

here, α and β are constants, we fix α=1.8, β=3.0 throughout this work. The piecewise function $g(x)$ is expressed as x_l^* ($x<x_l^*$), x ($x_l^*\leq x\leq x_u^*$), and x_u^* ($x>x_u^*$), where x_l^* and x_u^* are the lower and upper thresholds, respectively (Fig.1(A)). As shown in Fig.1(B), the effective potential generated by the thresholding is bistable with stable energy states at $x_-=\beta x_l^*/\alpha$ and $x_+=\beta x_u^*/\alpha$, in the absence of the input signals I_1 and I_2. By simply changing the threshold x_l^* and x_u^* in $g(x)$, we can vary the nonlinear characteristics of the dynamical system, and thus manipulate the heights and asymmetry of the potential wells.

The N coupled logic gates system can be given by the Langevin-type equation

$$\dot{x_i} = F(x_i) + I_1(t) + I_2(t) + D\eta_i(t) + \varepsilon\sum_{j=1}^{N} a_{ij}(x_j - x_i), \quad i,j = 1,\ldots,N, \tag{3}$$

The last term in Eq.(3) is the coupling term, in which ε is the coupling strength and a_{ij}=0 or 1 is the element of coupling matrix. For simplicity, we take the coupling

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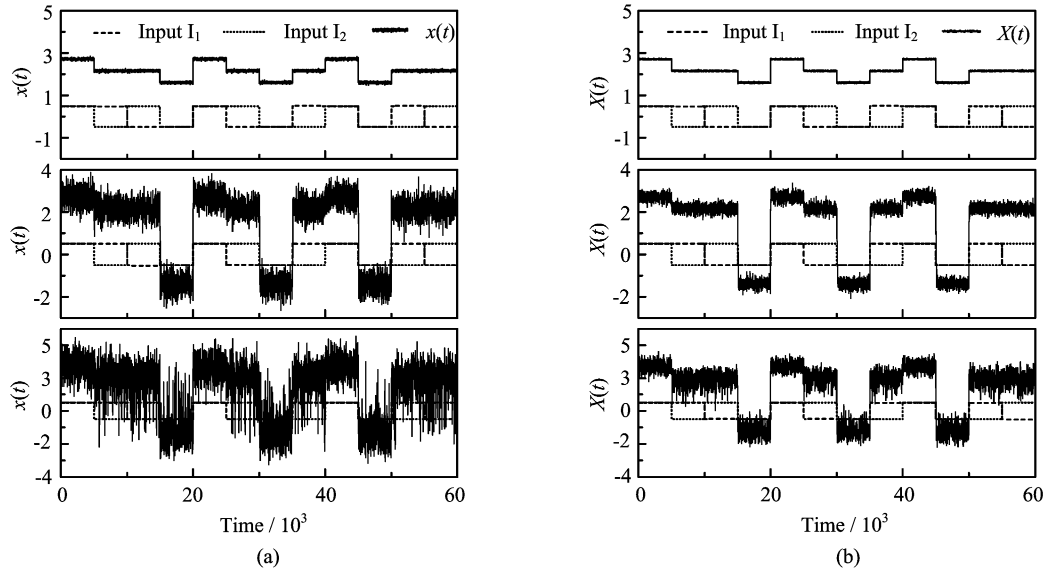

FIG. 2 The logic response (solid line) of (a) single logic gate and (b) coupled logic gates system (N=5), with thresholds (x_l^*, x_u^*)=(−0.5, 1.3). From top to down, the noise intensity D equals to 0.05, 0.6, and 1.0. The input sets consist of a combination of I_1 (dashed line) and I_2 (dotted line).

term as

$$\varepsilon\sum_{j=1}^{N}a_{ij}(x_j-x_i)=\varepsilon(x_{i+1}+x_{i-1}-2x_i) \qquad (4)$$

which means that a logic gate only couples to the nearest two gates. The collective response of the coupled system $X(t)$ is defined as

$$X(t)=\frac{1}{N}\sum_{i=1}^{N}x_i(t) \qquad (5)$$

To obtain the response of single logic gate and the coupled system, we integrate Eq.(1) and Eq.(3) by using Euler method with a proper time step length, respectively.

In this work, we use similar experiment setup in the Ref.[4] for each logic gate of the coupled system. Usually, a logical input-output association is obtained by encoding logic inputs in squares waves. Specially, for a two logic inputs, the input signal $I(t)$ is the sum of two aperiodic square waves,

$$I(t)=I_1(t)+I_2(t) \qquad (6)$$

with I_1 and I_2 encoding the two logic inputs. With no loss of generality, we set the two inputs $I_1(t)$ and $I_2(t)$ to take value −0.5 when the logic input is 0, and value 0.5 when logic input is 1. The logic input can be either 0 or 1, giving rise to four distinct logic input sets: (0, 0), (0, 1), (1, 0), and (1, 1). Since the logic input sets (0, 1), (1, 0) yield the same I value, the four distinct logic input sets can be encoded to three distinct values of $I(t)$, *i.e.*, the input signal $I(t)$ is a three-level aperiodic square wave [4]. The output of the bistable system can be decoded from the final state of the bistable dynamical system, when the system stays in one well, the output can be regarded as a logical 0 (or 1), and a logical 1 (or 0) when it stays in the other well. By modulating the asymmetry of the wells appropriately, we can achieve the desired logic outputs by driving the state of the system to one or the other well. For a given logic input set (I_1, I_2), a logic output from the nonlinear system can be checked based on the truth tables of the basic logic operations shown in Table I.

TABLE I Relationship between two logic inputs and the logic output of the four fundamental AND, NAND, OR, and NOR gates. The combination of the four fundamental logic gates can be used to construct any logic circuit.

Input set (I_1, I_2)	OR	NOR	AND	NAND
(0, 0)	0	1	0	1
(0, 1)/(1, 0)	1	0	0	1
(1, 1)	1	0	1	0

III. RESULTS AND DISCUSSION

The effects of coupling on the LSR phenomenon and the reliability of the logic system are presented. Here we start from single logic gate. In Fig.2(a), we show the responses $x(t)$ of the single logic gate (Eq.(1)) for thresholds (x_l^*, x_u^*)=(−0.5, 1.3), and from top to down the noise intensity D equals to 0.05, 0.6, and 1.0. We can observe that, under moderate noise intensity (*e.g.*, D=0.6), interpreting the state x>0 as logic output 0

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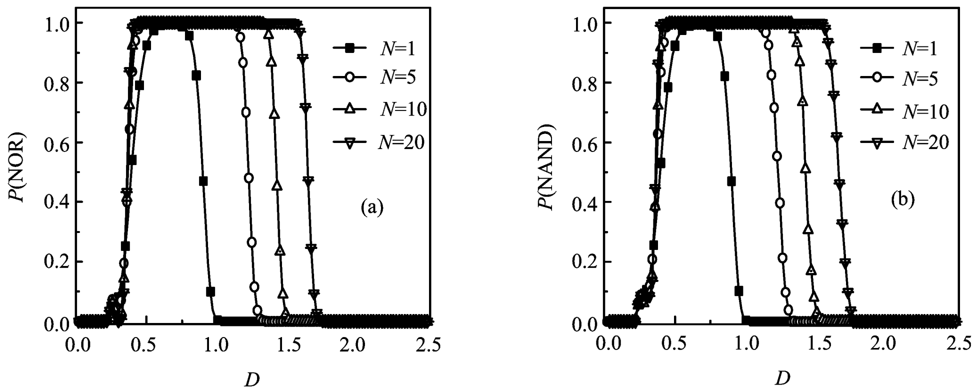

FIG. 3 (a) For the NOR logic operation, the success probability P(NOR) versus noise intensity D for single logic gate and three coupled logic gates systems (N=1, 5, 10, and 20) is shown, with (x_l^*, x_u^*)=(−0.5, 1.3). (b) For the NAND logic operation, similar results are obtained by taking (x_l^*, x_u^*)=(−1.3, 0.5). The coupling strength is ε=0.1. The results of P are obtained from numerical simulations over 10^5 different runs.

and the state x<0 as logic output 1 yields a stable logical NOR gate whereas interpreting the state x>0 as logic output 1 and the state x<0 as logic output 0 yields a stable logical OR gate. When D is too small (D=0.05) or too large (D=1.0), wrong logic outputs appear, and the logic gate starts to work unreliably. In a completely analogous way, by setting the thresholds at (x_l^*, x_u^*)=(−1.3, 0.5), we can realize clean AND/NAND gates in almost the same optimal noise intensity regime as the previous case. Note that NOR and NAND gates are the fundamental gates which can, in combination, realize all possible logical responses. Next, we turn to the coupled system. In Fig.2(b), We display the collective responses $X(t)$ for different D with system size N=5. Other parameters are the same as in Fig.2(a). When D=0.05, just like the single gate, the collective responses cannot produce desired logic output reliably. As the noise intensity is increased (D=0.6), the collective responses can represent a clear NOR operation. Interestingly, when the noise intensity is further increased to D=1.0, the coupled logic gates system still works very reliably, which is distinct from single gate. From Fig.2, we intuitively find that coupling can remarkably affect the reliability of the logic system.

In order to quantify the reliability of the logic system, we calculate the success probability of obtaining the desired logic outputs for different input sets. The success probability P(logic) is defined as the ratio of the number of correct logic outputs to the total number of runs, where each run samples over the four input sets (0, 0), (0, 1), (1, 0), (1, 1), in different permutations. Only when the logic outputs, obtained from $x(t)/X(t)$, match the logic outputs in the truth tables for all four input sets in the run, it is considered a success, and a failure otherwise [4]. When P(logic) is close to 1, the logic system is considered to work very reliably.

Figure 3 shows the success probability P(logic) as a function of D for single logic gate and three coupled logic gates systems (N=5, 10, and 20). Firstly, we notice that, for single logic gate (N=1), it is evident that the fundamental logic operations NOR (Fig.3(a)) and NAND (Fig.3(b)) are accurately realized (P(logic)∼1) in a moderate noise range, respectively. For very small or large noise the system cannot yield any consistent logic output (P(logic)≪1), which is in line with the basic tenets of stochastic resonances, thus this phenomenon is termed as LSR [4]. As for the coupled systems, the results in Fig.3 illustrate that the LSR phenomenon still exists robustly. More interestingly, compared with single logic gate, the coupled systems could yield reliable logic output in a much wider noise region. We notice that when N=1 the logic output can stay reliable (P(logic)∼1) from D=0.55 to 0.75. Whereas for coupled system (*e.g.*, N=5), under D=0.45−1.15, the logic output can be almost 100% accurate, which is in accordance with the visual inspection of Fig.2(b). For convenience, the noise region in which logic output can be obtained reliably is defined as reliable region. These observations demonstrate that the reliable region of coupled system is much wider than single gate, that is to say, coupling can remarkably improve the reliability of the logic system and thus enhance the LSR phenomenon. Following the common terminology, this behavior can be termed as array-enhanced logical stochastic resonance. It is also found that as the size of the coupled system increases, the reliable region becomes wider (see the curves of N=1, 5, 10, and 20 in Fig.3).

To quantitatively characterize the phenomenon of array-enhanced logical stochastic resonance, we introduce an order parameter L, which is defined as the size of the reliable region where the P(logic) is close to 1 (we choose P>0.97). The dependence of L on system size N is plotted in Fig.4. It can be observed that when N=1 (single logic gate), L is relatively small (about 0.2), as the system size N increases, the region size L initially increases rapidly, then slowly increases, and finally reaches a plateau, which is in accordance with the results displayed in Fig.3. The figure illustrates

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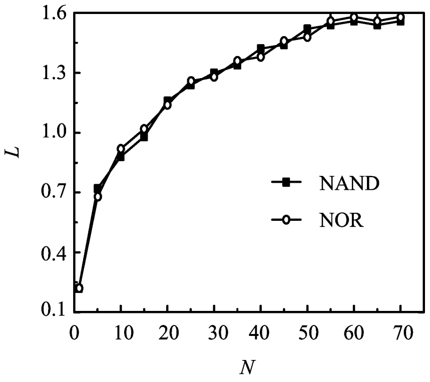

FIG. 4 Dependence of the reliable region size L on system size N for the NOR and NAND logic operation. Other parameters are the same as in Fig.3.

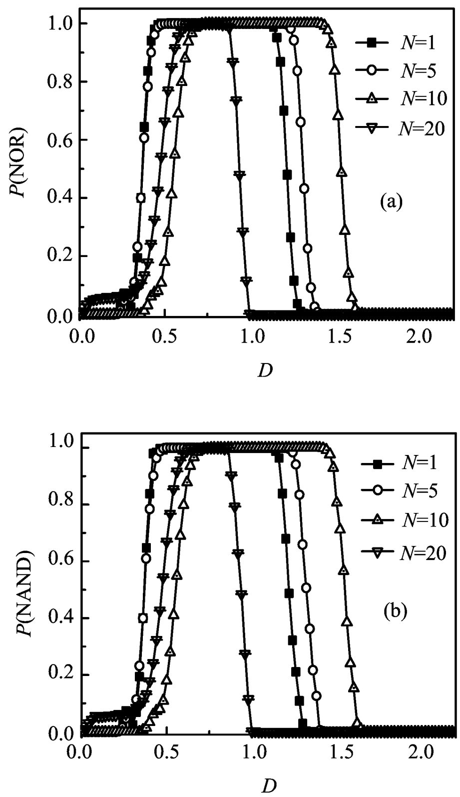

FIG. 5 The success probability P(NOR) versus noise intensity D for different coupling strength ε, with (x_l^*, x_u^*)=(−0.5, 1.3). (b) For the NAND logic operation, similar results are obtained by taking (x_l^*, x_u^*)=(−1.3, 0.5). The system size is N=5. The results of P are obtained from numerical simulations over 10^5 different runs.

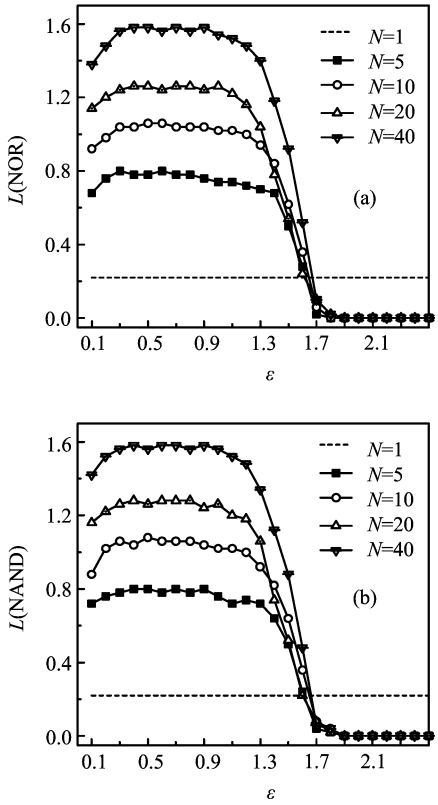

FIG. 6 (a) For the NOR logic operation, the size of the reliable region L as a function of coupling strength ε for different system size is shown. (b) For the NAND logic operation, similar results are obtained. Other parameters are the same as in Fig.5.

clearly that coupling can drastically improve the reliability of the logic system and enhance the LSR effect. The enhancement is larger for larger N, whereas for large enough size (*e.g.*, N>50), the enhancement seems to be saturated. It is noteworthy that for the NOR and NAND logic operation, the qualitative results are similar, but with some tiny quantitative differences.

We also consider the effect of coupling strength ε on the LSR phenomenon and the reliability of the coupled logic system. In Fig.5, P(logic) as a function of D is plotted for four different coupling strength ε, and the system size is fixed at N=5. When ε=0.1, the reliable region is about D=0.45−1.15, as ε is further increased, *e.g.*, to 0.3, the reliable region becomes wider. When we further increase ε to 0.5, it can be found that the reliable region shifts to higher D levels, whereas the size seems to be unchanged. Finally, when ε is larger (*e.g.*, ε=1.6), we can see that the reliable region decreases obviously. These observations indicate that the reliable region of the coupled system evolves non-monotonically as ε increases.

To qualify the impact of ε, the size of the reliable region L versus ε for different system size is plotted in Fig.6. Therein, the dashed line represents L for single logic gate. It is shown that for a fixed size of the coupled system, *e.g.*, N=5, as ε increases, L first increases, then keeps almost stable for a large range of ε, and finally decreases again. The results clearly present that the size of L can evolve non-monotonically as ε increases, indicating a typical resonance-like behavior. For different N, we can achieve similar qualitative results. Further-

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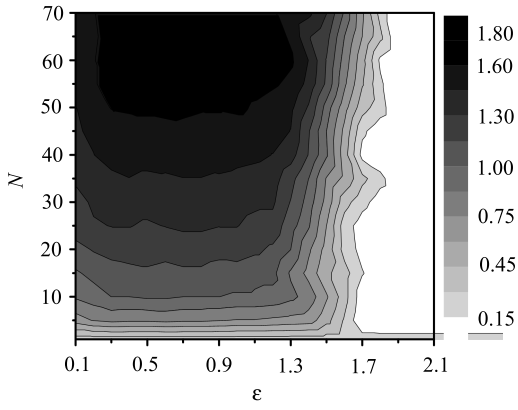

FIG. 7 Contour plot of L in dependence on the system size N and the coupling strength ε for the NOR logic operation.

more, it can be observed that in a wide range of ε (from 0.1 to 1.6), the L-ε curves of coupled logic systems are much higher compared with single logic gate (dashed line), which means that coupling can improve the reliability of the logic system and enhance the LSR effect in a considerable range of coupling strength. However, for strong enough coupling, *e.g.*, ε>1.7, L decreases to 0, the reliable region disappears, indicating that the phenomenon of array-enhanced logical stochastic resonance cannot exist when the coupling is too strong.

Finally, to make an overall inspection, the dependence of L on both system size N and coupling strength ε is shown in Fig.7. Since for NOR and NAND logic operation, the results are almost the same, for simplicity, we just show the results of NOR logic operation. In accordance with the results above, we can find coupling can improve the reliability of the logic system and thus enhance the LSR effect in a considerable plane of (N, ε). Whereas when the coupling strength is too large, the array-enhanced logical stochastic resonance is destroyed.

IV. CONCLUSION

In this work, the impact of coupling on the reliability of the logic system as well as the LSR phenomenon has been investigated in the coupled logic gates system. We find that for the coupled system, the LSR effect still exists robustly, *i.e.*, the system can produce desired logic output reliably in a moderate noise region (reliable region). Furthermore, it shows that the reliable region of coupled system is much wider than the single logic gate, which means coupling can obviously improve the reliability of the logic system and thus enhance the logical stochastic resonance. This enhancement is larger for larger system size, whereas for large enough size, the enhancement seems to be saturated. We also examine the effect of coupling strength, it can be observed that the size of the reliable region displays a resonance-like effect as a function of coupling strength. In addition, when the coupling is too strong, the reliable region disappears, which means that the behavior of array-enhanced logical stochastic resonance can be destroyed when the coupling strength is too large.

The concept of LSR proposed by Ditto's team has provided a feasible technology, in which device noise can be exploited to construct a reliable logic gate. On the other hand, in real computational devices, *e.g.*, integrated circuit (IC), a large number of logic gates are packed with conducting wires into the surface of semiconductor material to carry out complex computation. Coupling between logic gates is a very important factor in constructing computational devices. Our results may be instructive to understand that how coupling affect reliability of the logic system as well as the LSR phenomenon, and we hope that such findings would be helpful for design and development of future computational devices which can work reliably under the impact of thermal noise.

V. ACKNOWLEDGMENTS

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