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High-level Stark Effect and Spectrum of Spherical Nanometer System

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In the electric field and layer-to-layer interaction energy, the law of split-level of high-level Stark effect of spherical nanometer system is explored as well as the frequency of spectrum, intensity and size effect of coefficient of spontaneous radiation. Taking three layers CdS/HgS spherical nanometer system as an example, the influence of the electric field and layer-to-layer interaction energy is explored on Stark effect and spectrum. The results show that in the Stark effect system, the energy level is split based on 1, 3, $\cdots$, $(2n-1)$, when it is in the electric field only, similar to the hydrogen atoms; and in the electric field and layer-to-layer interaction, it is split based on 1, 4, $\cdots$, n^2; with the quantum transition, the frequency of the spectrum decreases with the increasing size of the system; apart from a few spectral lines, the intensity of most spectral lines will decreased as the size increases; while the coefficient of spontaneous radiation will increase with the increasing size; the electric field will cause the changes of spectrum frequency; its spectrum frequency shift is proportional to the square of the electric field intensity; apart from a few spectral lines, the frequency shift of spectral lines that is caused by the electric field and layer-to-layer interaction will decrease as the size increases; the interaction will make the level of electronic energy level lower slightly (the order of magnitude is between 10^{-7}–10^{-9} eV), the slightly increased spectrum intensity and the slightly increased value of coefficient of spontaneous radiation, but it will not influence the frequency of spectrum, intensity, and the trend that coefficient of spontaneous radiation changes with the size; when the size is smaller, the layer-to-layer interaction effect will be significant.

Key words: Layer-to-layer interaction energy, CdS/HgS/CdS spherical nanometer system, Stark effect, Frequency shift of spectral line, Coefficient of spontaneous radiation

I. INTRODUCTION

The spherical nano-system and its Stark effect is very important to the development of physical theory, the development of sensors and other applications of new electronic components. So far, it has been a lot of researches on it. Ji *et al.* have studied quantum magnetic dynamics of polarized light in arrays of microcavities [1]. By adjusting the Zeeman and the Raman fields, they could realize a ferromagnetic phase, super-counter-fluidity phase, and antiferromagnetic phase of polarized light, that are of interest for studying spin-dependent photon-photon interactions. The dynamic creation of fractionalized half-quantum vortices in Bose-Einstein condensates of sodium atoms was also investigated [2]. The Josephson effect for photons in two weakly linked microcavities and non-Abelian Josephson effect between two F=2 spinor Bose-Einstein condensates in double optical traps were described in Refs.[3, 4]. For semiconductor nano-system, the theoretical and experimental researches on the energy spectrum of CdS/HgS/CdS spherical nano-system have also been made [5, 6]. The preparation of the closed-end CdS/HgS/CdS and the open HgS/CdS/HgS spherical nano-systems and the relationship between electric energy spectrum and size were given in Refs.[7, 8]. But these researches did not take the roles of layer-to-layer interaction energy and electric field into consideration.

By considered the layer-to-layer interaction energy, Zheng *et al.* studied the electric energy spectrum and probability distribution of HgS/CdS/HgS columnar nano-system with regarding the layer-to-layer interaction energy as a constant [9]. In electric field, quantum dots and quantum wires had Stark effect, and the Stark energy level excursion was related to the electric field and sample-size [10, 11]. The studies of the low-level Stark effect of spherical nano-system without considering the layer-to-layer interaction; and the relationship between the layer-to-layer interaction potential and size of spherical nano-system with Green function method and Fourier transformation were also carried out [12,

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13]. The influence of layer-to-layer interaction on Stark effect of spherical nano-system and spectrum is a very important problem to be solved.

Taking CdS/HgS/CdS spherical nano-system for example, in this work, we studied the influences of layer-to-layer interaction, electric field and size on the high-level Stark effect, the frequency and intensity of spectrum, and coefficient of spontaneous radiation.

II. ELECTRONIC ENERGY AND WAVE FUNCTION

Three-layer spherical nano-system consists of CdS, HgS, and CdS. The inner is CdS with radius named R_1 and the dielectric constant named ε_1. The mesosphere is HgS with radius named R_2, and the thickness $L=R_2-R_1$ is very thin, the dielectric constant is ε_2. The outside is CdS, and the radius is greater than R_2, the dielectric constant is $\varepsilon_3=\varepsilon_1$. Applied uniform electronic field E along the Z axis, at the same time taking the electric field and layer-to-layer interaction into consideration, the electronic potential is given by

$$\varphi(r) = V_0(r) + V(r) + U(r) \tag{1}$$

$V_0(r)$, $V(r)$, and $U(r)$ are electronic potential, electrostatic potential, and layer potential without considering the layer-to-layer interaction, respectively. When $0<r<R_1$, $r>R_2$, $V_0(r)=V_1$; while $R_1<r<R_2$, $V_0(r)=V_2$. $V(r)$ can be written as [13]

$$V(r,\theta,\varphi) \approx qE\left[Br+\frac{C}{r^2}\right]\cos\theta \tag{2}$$

$$B = \frac{(\varepsilon_3+2\varepsilon_2)C}{(\varepsilon_2-\varepsilon_1){R_1}^3} \tag{3}$$

$$C=\frac{3\varepsilon_1(\varepsilon_2-\varepsilon_1){R_1}^3{R_2}^3}{(\varepsilon_2+2\varepsilon_1)(\varepsilon_3+2\varepsilon_2){R_2}^3+2(\varepsilon_1-\varepsilon_2)(\varepsilon_2-\varepsilon_3){R_1}^3} \tag{4}$$

The relationship of $U(r)$ and r is:

when $r<R_1$,

$$U(r)\approx -\frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}^2}{2\varepsilon_1(r)R_1}\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\left\{\frac{{R_1}^2}{{R_1}^2-r^2}+\frac{\varepsilon_1}{\varepsilon_2}F\left[1,\gamma,\gamma+1,\left(\frac{r}{R_1}\right)^2\right]\right\} \tag{5}$$

$$\varepsilon_1(r)=\frac{\varepsilon_1+\varepsilon_2}{2}\left[1-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\mathrm{th}\left(\frac{r-R_1}{L}\right)\right] \tag{6}$$

When $R_1<r<R_2$,

$$U(r)\approx -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{2\varepsilon_2(r)R_2}\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_1+\varepsilon_2}\left\{\frac{{R_2}^2}{{R_2}^2-r^2}+\frac{\varepsilon_1}{\varepsilon_2}F\left[1,\gamma,\gamma+1,\left(\frac{r}{R_2}\right)^2\right]\right\} \tag{7}$$

$$\varepsilon_2(r)=\frac{\varepsilon_1+\varepsilon_2}{2}\left[1-\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2+\varepsilon_1}\mathrm{th}\left(\frac{R_2-r}{L}\right)\right] \tag{8}$$

When $R_2<r<\infty$,

$$U(r)\approx -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{2\varepsilon_3(r)R_2}\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_1+\varepsilon_2}\left\{\frac{{R_2}^2}{r^2-{R_2}^2}+\left(\frac{R_2}{r}\right)^2F\left[1,\gamma,\gamma+1,\left(\frac{R_2}{r}\right)^2\right]\right\} \tag{9}$$

$$\varepsilon_3(r)=\frac{\varepsilon_1+\varepsilon_2}{2}\left[1-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\mathrm{th}\left(\frac{r-R_2}{L}\right)\right] \tag{10}$$

here, $F(x,y,z,u)$ is the confluence hypergeometric function, the specific expression is seen in Ref.[14].

First, the electrostatic potential $V(r)$ is perturbed, and the electron energy that is not perturbed which is figured out with the separated variable method, then the electron energy $E^{(0)}_{n,l}$ and the wave function are

$$E^{(0)}_{n,l} = \frac{n^2\pi^2\hbar^2}{2\mu L^2}+\frac{l(l+1)}{2\mu {R_0}^2}\hbar^2+|V_1-V_2| \tag{11}$$

$$\psi^0_{n,l,m}=\phi^{(0)}_n(r)y_{l,m}(\theta,\varphi) \tag{12}$$

For $R_1<r<R_2$

$$\psi^0_{n,l,m}=\sqrt{\frac{2}{L}}\frac{1}{r}\sin\frac{n\pi(r-R_1)}{L}y_{l,m}(\theta,\varphi) \tag{13}$$

For $r<R_1$, $r>R_2$

$$\psi^0_{n,l,m}=0 \tag{14}$$

here, n=1, 2, $\cdots$; l=0, 1, $\cdots$, (n–1); m=0, ±1, $\cdots$, ±l, R_0 is the effective radius of gyration. The relationship of R_0, R_1, R_2 and the mass density ρ_1, ρ_2 of CdS and HgS can be seen in the Refs.[13, 15].

With perturbation method, the energy first-order correction that is caused by the electric field is zero, the second-order correction is given by

$$\Delta E^{(2)}_{n,l}=\frac{q^2E^2L^2}{48n^2E^{(0)}_{n,0}}\left|V(R_1,R_2)\right|^2(f_{n,l}+g_{n,l}) \tag{15}$$

The relationship of $V(R_1,R_2)$, $f_{n,l}$ and $g_{n,l}$ between L, R_0, and l,m can be seen in the Ref.[13]. When there is electric field, the electronic energy level and the wave function are

$$E_{n,l}\approx E^{(0)}_{n,l}+\Delta E^{(2)}_{n,l}$$
$$\psi_{n,l,m}=\psi^{(0)}_{n,l,m}+\frac{\mu{R_0}^2}{\hbar^2}\left(\frac{V_{l,l+1}}{l+1}\psi^{(0)}_{n,l+1,m}-\frac{V_{l,l-1}}{l}\psi^{(0)}_{n,l-1,m}\right) \tag{16}$$

Then, regarding the layer-to-layer interaction energy as perturbation, with the perturbation method, and taking the layer-to-layer interaction and electric field into

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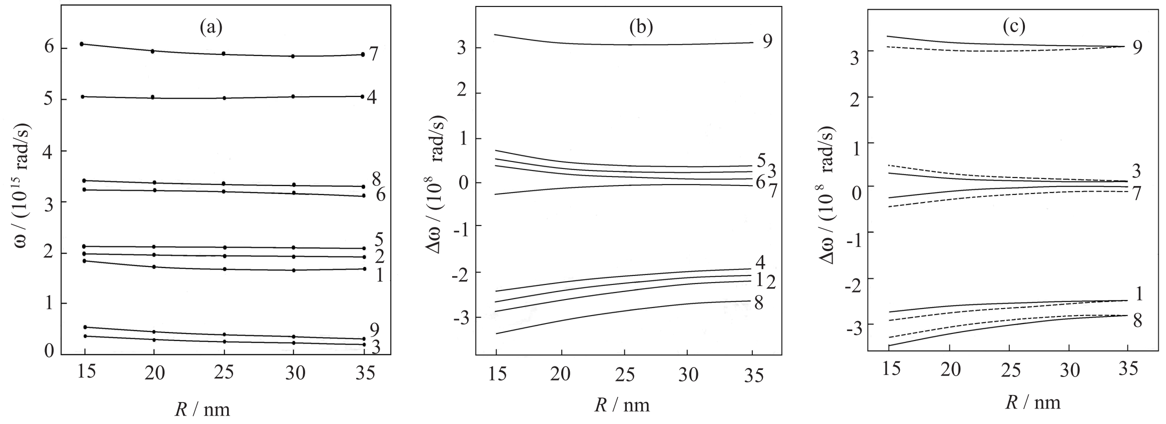

FIG. 1 The variation of the frequency ω and frequency shift $\Delta\omega$ of some spectral lines with size. The broken lines are the results caused by the electric field and layer-to-layer interaction, and the solid lines are the ones caused by only the electric field. (a) The variation of ω with size, (b) The variation of the $\Delta\omega$ caused by electric field with the size, (c) The variation of the $\Delta\omega$ caused by electric field and layer-to-layer interaction with the size. The digital signs in right number for spectral lines.

consideration, the electronic energy level

$$E \approx E_{n,l}^{(0)} + \Delta E_{n,l,m} \quad (17)$$

$$\begin{aligned}\Delta E_{n,l,m} &= E_{n,l}^{(2)} + E_{n,l,m}^{(3)} \\ &= E_{n,l}^{(2)} + \int \psi_{n,l,m}^{*} U(r) \psi_{n,l,m} \mathrm{d}\tau \end{aligned} \quad (18)$$

According to Eq.(17) and (18), the $\Delta E^{(2)}$ caused by electric field is only related to n, l, and the $E_{n,l,m}^{(3)}$ caused by layer-to-layer interaction is related to n, l, m.

III. SPECTRUM FREQUENCY

Arvchin *et al.* indicated [11], without electric field and layer-to-layer interaction, in the quantum transition, the spectrum frequency is $\omega = \hbar l/\mu R^2{}_0$. With electric field and inter-layer, the energy level shifted and caused the frequency shift of spectral line $\Delta\omega$. When n and l are given,

$$\Delta\omega = \frac{1}{\hbar}\left[\Delta E_{n,l+1}^{(2)} + \Delta E_{n,l-1}^{(2)} + \Delta E_{n,l+1,m}^{(3)} + \Delta E_{n,l-1,m}^{(3)}\right] \quad (19)$$

According to Eq.(19), ω depends on the effective radius of gyration R_0, while the frequency shift of spectral line $\Delta\omega$ is related to the electric field, layer-to-layer interaction, and linear.

According to Eq.(15), $\Delta E_{n,l}^{(0)}$ is proportional to the square of electric field intensity E^2, thus the $\Delta\omega$ caused by electric field is proportional to E^2, too. According to Eq.(17) and Eq.(18), because layer-to-layer interaction is unrelated to E, after considering the layer-to-layer interaction, the frequency shift of spectral line is still proportional to E^2.

The results reported in Refs.[11, 16] showed, for CdS, a_{CdS}=0.5818 nm, the permittivity $\varepsilon_{\mathrm{CdS}}$=9.1, the mass density $\rho_{\mathrm{CdS}}=4.82\times10^3$ kg/m^3, the electronic effective mass $\mu_{\mathrm{CdS}}=0.2m_0$, ($m_0$ is electronic rest mass), $V_1=-80$ meV; for HgS, the corresponding values are a_{HgS}=0.5851 nm, $\varepsilon_{\mathrm{HgS}}$=18.2, $\rho_{\mathrm{HgS}}=8.10\times10^3$ kg/m^3, $\mu_{\mathrm{HgS}}=0.036m_0$, $V_2=-1.2$ eV. If $L=R_2-R_1$=5 nm, E=20 kV/cm, taking different values of R_1 and R_2 into Eq.(2), the values of B and C can be obtained. Taking different values of n_1, n_2, and the obtained values of B, C into Eq.(13), $\Delta E_{n,l}^{(2)}$ is figured out. According to $\omega=\hbar l/\mu R_0^2$, Eqs.(17)–(19), ω and $\Delta\omega$ are obtained. While the spectrum frequency ω without electric field, the frequency shift $\Delta\omega$ caused by electric field, and the $\Delta\omega$ caused by electric field and layer-to-layer interaction changing with size are shown in Fig.1. The nine spectral lines are: 1($|210\rangle\rightarrow|100\rangle$), 2($|211\rangle\rightarrow|100\rangle$), 3($|21\text{-}1\rangle\rightarrow|100\rangle$), 4($|310\rangle\rightarrow|100\rangle$), 5($|311\rangle\rightarrow|100\rangle$), 6($|31\text{-}1\rangle\rightarrow|100\rangle$), 7($|321\rangle\rightarrow|210\rangle$), 8($|321\rangle\rightarrow|100\rangle$), 9($|32\text{-}1\rangle\rightarrow|100\rangle$).

The figure shows that: all the spectrum frequencies decreased with the increasing size (see Fig.1(a)), the $\Delta\omega$ caused by electric field is related to the size, $\Delta\omega$ of most spectral lines will decrease with increasing size, while the opposite is true for a small number of spectral lines (see Fig.1(b)); $\Delta\omega$ caused by layer-to-layer interaction of some spectral lines will decrease with increasing size, while the opposite is true for some spectral lines, but it does not influence the trend of ω's variety with size. When the size is smaller, its influence is significant (see Fig.1(c)). Comparing the experiment of Ref.[11] and Fig.1, it shows that $\Delta\omega$ caused by electric field changes with R_1, the changing trend of CdS/HgS/CdS nano-system is similar to that of CdS quantum dots, and the similar magnitude. The experiment shows [11]: when E=12.5 kV/cm, R_1=15 nm,

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R_2=20 nm, the spectral lines frequency shit of spherical nano-system in CdS/HgS/CdS: $\Delta\omega$=6 GHz, then, $\Delta E\approx$24.8 μeV. In this work, $\Delta E\approx$16.6 μeV, it shows the values of theoretical calculations are close to those of experiment.

IV. SPECTRUM INTENSITY

According to Einstein's theory of spontaneous radiation, the intensity ratio for the spectral line α of the frequency ω_α and the spectral line β of the frequency ω_β is

$$\frac{I_\alpha}{I_\beta}=\frac{{\omega_\alpha}^4}{{\omega_\beta}^4}\frac{|\mathbf{r}_\alpha|^2}{|\mathbf{r}_\beta|^2} \quad (20)$$

here, $\omega_{k'k}$ is the spectrum frequency of k' state$\rightarrow k$ state, k=n,l,m, k'=n',l',m', $\mathbf{r}_{nlm,n'l'm'}=\int\psi^*_{nlm}\mathbf{r}\psi_{n'l'm'}\mathrm{d}\tau$.

Taking Eq.(16) into the equation above, the matrix elements are obtained, according to the Eq.(20), the intensity ratio for spectral lines i and the spectral line 1 can be obtained. The results shows: the spectral line meets transition Δl=0, its intensity is 0. When the intensity is not 0, only Δl=±1 is taken, that is, it also meets the selection rules of the electric dipole radiation. The intensity of the first spectral line as a standard with taking electric field and layer-to-layer interaction into account, the spectral line I_i/I_1 changes with the size (Fig.2). It shows that the intensity of some spectral lines (such as the spectral lines 3, 4, 5, and 7) will decrease as the size increase. While the opposite is true for some spectral lines for example, the spectral line 8; the layer-to-layer interaction does not change the trend that I_i/I_l of spectral lines variety with the size. However, it will make the I_i/I_1 value of the spectral line slightly increase, and when the size is smaller, the effect of layer-to-layer interaction will be significantly increased.

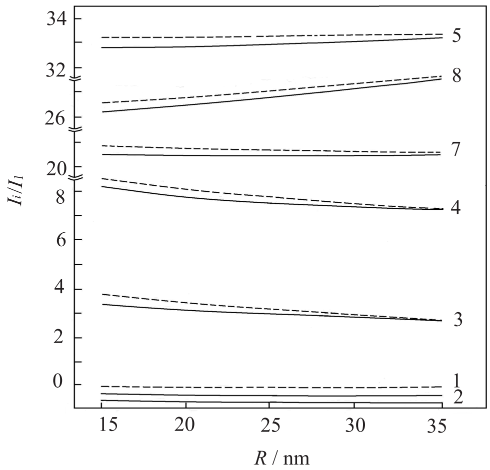

FIG. 2 The variation of the relative intensity of part spectral lines with size, the situation caused by electric field is represented with the solid lines, the situation caused by electric field and layer-to-layer interaction is represented with the broken lines. The digital signs in right number for spectral lines.

V. THE ENERGY LEVEL SPLIT LAW OF STARK EFFECT

The low energy level split law of Stark effect of HgS/CdS/HgS spherical nanometer system was given in Ref.[12]. The universal law is explored here.

According to Eq.(11), the energy level under zero-order approximation is the degeneracy in the system. In accordance with the degeneracy perturbation theory, amendment to the first-order correction of the energy is decided by the following equation:

$$\det\|H'_{li}-E'_1\delta_{li}\|=0 \quad (21)$$

$$H'_{li}=\int\phi^*_l\hat{H}'\phi_i\mathrm{d}\tau \quad (22)$$

$$\delta_{li}=1 \quad (l=i) \quad (23)$$

$$\delta_{li}=0 \quad (l\neq i) \quad (24)$$

By orthogonal normalized quality of the spherical harmonics, only when it meets l=l'±1, m=m', H'_{li} is not equal to zero,

$$\begin{aligned}H'_{l(l+1)}&=\int\phi^*_lH'\phi_{l+1}\mathrm{d}\tau\\&=\int\psi^*_{n(l+1)m}qE\left(Br+\frac{C}{r^2}\right)\cos\theta\psi_{nlm}\mathrm{d}\tau\\&=H'_{(l+1)l}\end{aligned} \quad (25)$$

when m=±m', the conjugate of ψ_{nlm} and $(-1)^m\psi_{nl,-m}$ with the same n,l. According to this, the corresponding matrix element with ±m' is

$$\begin{aligned}H'_{l(l+1),m}&=\int\psi^*_{nl,-m}qE\left(Br+\frac{C}{r^2}\right)\cos\theta\psi_{n(l+1),-m}\mathrm{d}\tau\\&=H'_{(l+1)l,-m}\end{aligned} \quad (26)$$

By the types Eqs.(25) and (26), the corresponding matrix elements of the energy levels can be obtained.

Easily, first-order correction of the ground state energy is $E^{(1)}_1$=0, that is, the ground state energy level is not split. When n=2, the matrix element H'_{12}=H'_{21}=$-(3/16)BqEa_0$, a_0 is the Bohr radius of the first orbit, its matrix element is zero. The equation is as follows:

$$\begin{vmatrix}-E^{(1)}_2 & H'_{12} & 0 & 0\\ H'_{12} & -E^{(1)}_2 & 0 & 0\\ 0 & 0 & -E^{(1)}_2 & 0\\ 0 & 0 & 0 & -E^{(1)}_2\end{vmatrix}=\begin{vmatrix}A_2 & 0\\ 0 & A_1\end{vmatrix}=0 \quad (27)$$

here, A_1 and A_2 is the determinant of the "2×2" block matrices of square, its product A_1A_2=0. Thus, solving

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the Eq.(27), $E_2^{(1)}$=0, 0, $\pm(3/16)BqEa_0$, 2 repeated root can be obtained. Therefore, when n=2, the energy level is split to N=2×2–1=3.

When n=3, the electronic energy level of non-perturbation is 9 degeneracy, and the matrix element is

$$H'_{12}=H'_{21}=-\frac{3\sqrt{6}}{16}BqEa_0 \qquad (28)$$

$$H'_{23}=H'_{32}=-\frac{3\sqrt{3}}{16}BqEa_0 \qquad (29)$$

$$H'_{45}=H'_{54}=H'_{67}=H'_{76}=-\frac{9}{32}BqEa_0 \qquad (30)$$

the rest are zero. The equation is as follows:

$$\begin{vmatrix} -E_3^{(1)} & H'_{12} & & & & & & & \\ H'_{12} & -E_3^{(1)} & H'_{23} & & & & & & \\ & H'_{23} & -E_3^{(1)} & 0 & & & & 0 & \\ & & 0 & -E_3^{(1)} & H'_{45} & & & & \\ & & & H'_{45} & -E_3^{(1)} & 0 & & & \\ & & & & 0 & -E_3^{(1)} & H'_{45} & & \\ & & 0 & & & H'_{45} & -E_3^{(1)} & 0 & \\ & & & & & & 0 & -E_3^{(1)} & 0 \\ & & & & & & & 0 & -E_3^{(1)} \end{vmatrix}$$

$$=\begin{vmatrix} A_3 & 0 & 0 & 0 \\ 0 & A_2 & 0 & 0 \\ 0 & 0 & A_{-2} & 0 \\ 0 & 0 & 0 & A_1 \end{vmatrix}=0 \qquad (31)$$

here, A_3 is the determinant of order "3×3", A_2, A_{-2}, and A_1 is the determinant of order "2×2", and its product $A_1A_{-2}A_2A_3$=0. Thus, the answer of equation is:

$$E_3^{(1)}=0,0,0,\pm\frac{9}{32}BqEa_0,\pm\frac{9}{32}BqEa_0,\pm\frac{9}{16}BqEa_0$$

The energy levels are split to N=2×3–1=5.

By Eq.(27) and the equation above, the equations all consist of the block matrices of square determinant. Because when $m\neq m'$, the matrix element of $H'_{l(l+1)}$ is zero, the block matrices of square determinant is divided from $m\neq m'$, and all elements are corresponding to a value m within $m\neq m'$. If there are i functions, there will be i row-column in the block matrices of square determinant.

Similarly, the equation as follows for the arbitrary n is given:

$$\begin{vmatrix} A_n & & & & & & & & & \\ & A_{n-1} & & & & & & & & \\ & & A_{-(n-1)} & & & & & & & \\ & & & \ddots & & & 0 & & & \\ & & & & A_{n-m} & & & & & \\ & & & & & A_{-(n-m)} & & & & \\ & 0 & & & & & \ddots & & & \\ & & & & & & & A_2 & & \\ & & & & & & & & A_{-2} & \\ & & & & & & & & & A_1 \end{vmatrix}=0 \qquad (32)$$

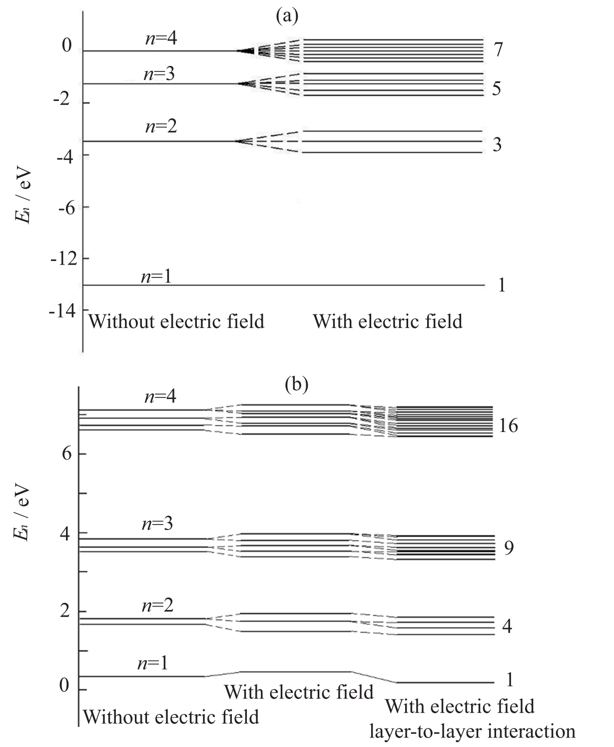

FIG. 3 The energy level splitting graphs of Hydrogen atom and the spherical nanometer system. (a) Hydrogen atom and (b) the spherical nanometer system. The digital signs in right are numbers of spectral lines.

where A_n is the determinant of the "$n\times n$" block matrices of square and it meets the need $A_{n-1}=A_{-(n-1)}$. A_{n-1} is the determinant of the "$(n-1)\times(n-1)$" block matrice of square and meets $A_{n-1}=A_{-(n-1)},\ldots$; A_2 is that of "2×2" and $A_2=A_{-2}$. A_1 is the determinant of the single row-column block matrice of square. According to Eq.(32) diagonal symmetry, except for $E_n^{(1)}$ has zero solution in A_1, while $E_n^{(1)}$ has non-zero solution in $A_{\pm2}$, $A_{\pm3}$, $A_{\pm4}$, $\cdots$, and multiple roots, that is, degeneracy. A_n has 2 different real roots with non-zero solutions. Therefore, the energy levels of n^2 degeneracy will be split to $N=2n-1$ ones.

In short, the energy level split law of first-order Stark effect of spherical nano-system is: the energy level of n=1 can not be split; that of n=2 can be split into 3 sub-levels, $\cdots$; n energy levels can be split into 2n–1 sub-levels, that is, the energy levels are split by 1, 3, $\cdots$, 2n–1, similar to the hydrogen atoms.

According to Eq.(17) and Eq.(18), the energy level split situations in electric field can be shown in the Fig.3(b), obviously different from those of hydrogen atoms shown in the Fig.3(a).

After considering electric field and layer-to-layer interaction, by the Eq.(17) and Eq.(18), the degeneracy of energy level is totally eliminated. n energy levels can be split into n^2 ones. The energy level split situations

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TABLE I The variation of the relative coefficient of spontaneous radiation A_i/A_1 with the size in electric field.

R_1/nm	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6	Line 7	Line 8	Line 9
15	0.022428474	1.1616×10^{-10}	28.4991	0.06752	0.72356	1.3351×10^{-9}	0.13673	0.31766	2.5506×10^{-6}
20	0.022428474	0.1786×10^{-10}	57.1406	0.06776	0.95373	0.5667×10^{-9}	0.13699	0.31846	0.5450×10^{-6}
25	0.022428474	0.0404×10^{-10}	86.1386	0.06777	0.96308	0.3960×10^{-9}	0.13739	0.32159	0.1795×10^{-6}
30	0.022428474	0.0090×10^{-10}	172.704	0.06785	0.97032	0.1060×10^{-9}	0.13775	0.32317	0.0641×10^{-6}
35	0.022428474	0.0023×10^{-10}	177.618	0.06794	0.97692	0.0341×10^{-9}	0.13791	0.32502	0.0285×10^{-6}

TABLE II The variation of the relative coefficient of spontaneous radiation A_i/A_1 with the size in electric field and layer-to-layer interaction.

R_1/nm	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6	Line 7	Line 8	Line 9
15	0.022428463	0.5221×10^{-10}	33.2726	0.07686	0.72969	0.0189×10^{-9}	0.14492	0.359954	3.2988×10^{-6}
20	0.022428464	0.6853×10^{-10}	63.9806	0.07501	0.73278	0.3266×10^{-9}	0.15422	0.368875	1.3145×10^{-6}
25	0.022428464	0.9363×10^{-10}	92.3371	0.07307	0.74999	0.6658×10^{-9}	0.15598	0.373254	0.5386×10^{-6}
30	0.022428464	1.1733×10^{-10}	176.979	0.07130	0.75848	1.0499×10^{-9}	0.15798	0.377415	0.1710×10^{-6}
35	0.022428465	1.3985×10^{-10}	353.105	0.06954	0.76645	1.3287×10^{-9}	0.15992	0.381842	0.0142×10^{-6}

are shown in Fig.3(b).

Figure 3 and calculation shows that: the layer-to-layer interaction can not only make the energy levels be split into much more, but also can make all the energy levels shift, however, the amount of the energy level shift is not large (the order of magnitude is 1−100 neV). To the energy levels of n=1, 2, 3, 4, only considering the effect of electric field, the energy levels can be split into 1, 3, 5, 7, and 16 sub-levels altogether. At the same time, and considering the effect of layer-to-layer interaction, they can be split into 30 sub-levels. According to the selection rules of $\Delta l=\pm1$ above, the energy levels will transit to the low levels conditions of n=4. The numbers of the spectral lines are 33 only under electric field; while the number of the spectral lines will be 85 after taking both electric field and layer-to-layer interaction into account. Thus, the effect of layer-to-layer interaction can make the number of spectral lines more.

VI. COEFFICIENT OF SPONTANEOUS RADIATION

In accordance with Einstein's spontaneous radiation theory, when the system state transitions from k state to k' state, the coefficient of spontaneous radiation is:

$$A_{kk'} = \frac{4e^2\omega_{kk'}^3}{3\hbar c}\left|\mathbf{r}_{kk'}\right|^2 = \frac{4e^2\omega_{kk'}^3}{3\hbar c}\left(\left|x_{kk'}\right|^2+\left|y_{kk'}\right|^2+\left|z_{kk'}\right|^2\right) \quad (33)$$

$$x_{kk'} = \iiint \psi_{nlm}^* r^3 \sin^2\theta\cos\varphi\psi_{n'l'm'}\mathrm{d}r\mathrm{d}\theta\mathrm{d}\varphi \quad (34)$$

$$y_{kk'} = \iiint \psi_{nlm}^* r^3 \sin^2\theta\sin\varphi\psi_{n'l'm'}\mathrm{d}r\mathrm{d}\theta\mathrm{d}\varphi \quad (35)$$

$$z_{kk'} = \iiint \psi_{nlm}^* r^3 \sin\theta\cos\varphi\psi_{n'l'm'}\mathrm{d}r\mathrm{d}\theta\mathrm{d}\varphi \quad (36)$$

Taking the wave functions only in electric fields and the ones in electric field and layer-to-layer interaction respectively into the Eq.(33), $A_{kk'}\neq0$ can be obtained under the conditions of $\Delta l=l-l'=\pm1$, and it is equal to zero under other conditions. The ratios of A_i/A_1 narrate spectral lines for the coefficient of spontaneous radiation only in electric field and the coefficient A_1 of the first spectral line are shown in Table I, the ratios of A_i/A_1 in electric field and layer-to-layer interaction are shown in Table II.

According to Table I and Table II, only with the effect of the electric field, the coefficient of spontaneous radiation of most spectral lines (such as the spectral lines 3, 4, 5, 7, 8) will increase with the size; and the opposite is true for a few spectral lines (such as the spectral lines 2, 6, 9), and the coefficient of these spectral lines is very small ($A_i/A_l<10^{-6}$). The effect of layer-to-layer interaction does not change the trend that the coefficient of spontaneous radiation of the spectral lines variations with the size, but it can make the value slightly increase.

VII. CONCLUSION

Stark effect can appear in the spherical nano-system, but the amount of energy level shift is related to the electric field, the system size and the layer-to-layer interaction. When the size is smaller, the Stark effect will be obvious, when the size tends to infinity, the phenomenon of Stark effect will disappear.

When there exists the effect of electric field, the energy levels of Stark effect will be split in accordance with the law of 1, 3, $\cdots$, (2n-1), similar to the hydrogen atoms; when there exits the effect of electric field and layer-to-layer interaction, the energy levels of Stark effect will be split in accordance with the law of 1, 4, $\cdots$, n^2; when the spontaneous radiation transitions, it

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also meets the $\Delta l=\pm1$ selection rules of electric dipole radiation, energy levels will transit to the low levels conditions of $n=4$; The number of the spectral lines are 33 sub-levels only under electric field; while the number of the spectral lines will be 85 sub-levels after taking both electric field and layer-to-layer interaction into account. Thus, the effect of layer-to-layer interaction can make the number of spectral lines more.

The spectrum frequency shift $\Delta\omega$ caused by electric field is proportional to the square of the electric field intensity; except a few of spectral lines, the spectrum frequency shift $\Delta\omega$ caused by electric field and layer-to-layer interaction will decrease as the linear increase.

The layer-to-layer interaction can make the electronic energy level decrease slightly (1–100 neV), but it does not influence the trend that the relative intensity, frequency and the coefficient of spontaneous radiation variations with the size; only when the values of the relative intensity and the coefficient of spontaneous radiation increase, and the size is smaller, the influence is obvious. The values of theoretical calculations are close to those of experiment results.

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